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A Methodology for the Choice of the Best Fitting Continuous-Time Stochastic Models of Crude Oil Price: The Case of Russia

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ABSTRACT: In this study, it has been attempted to select the best continuous- time stochastic model, in order to describe and forecast the oil price of Russia, by information and statistics about oil price that has been available for oil price in the past. For this purpose, method of The Maximum Likelihood Estimation is implemented for estimation of the parameters of continuous-time stochastic processes. The result of unit root test with a structural break, reveals that time series of the crude oil price is a stationary series. The simulation of continuous-time stochastic processes and the mean square error between the simulated prices and the market ones shows that the Geometric Brownian Motion is the best model for the Russian crude oil price.

Keywords: Stochastic processes; Crude oil price; Unit root test; Structural break; MLE estimation; Simulation
JEL Classifications: C51; C53

1. Introduction

Long term planning and budgeting are the main tools of the governments to reach to the self-ideal social and economic objectives. The correct prediction of main economic variables facilitates the way for collection of effective programs. The reliance of some governments on the oil revenues for finance security of its expenditures, lead to the price of oil and oil revenues, to be effective variables in the economic conditions of such countries. So by more accurate forecast about the future oil price, the politician can develop the effective and efficient programs. A lot of studies with different goals have engaged to inspect the oil price series manner. Some studies for forecasting of the oil price have been done reduce the manner of this time series. The goal of some other studies is the evaluation of oil deposit. Researches that have engaged in the evaluation of the oil resources, in addition to inspection of casually manner oil price, have done something for inspection of other variables manner such as convenience yield and risk-free interest rate. In this research putting other to importance on the oil incomes in government budgeting in oil-bearing countries become take action to find the best fitting continuous-time process to explain the oil price series manner.

Gibson and Schwartz (1989) have accepted the nonstationarity hypothesis for the crude oil price and have applied the Geometric Brownian Motion (hereafter, GBM) to evaluate the oil-linked assets. They assumed that the convenience yield and the interest rate are constant. The Augmented Dickey-Fuller and the variance ratio tests used by Pindyck (1999) for the crude oil price series have shown that a mean-reverting stochastic process can be applied for the oil price model. However, the reversion speed is slow. Pindyck (1999) offered that the equilibrium level is not constant over time. Dias and

Rocha (1999) detected that the large jumps, occur some weeks after abnormal events. At the time of these events, three jumps up and two jumps down can be observed. This encouraged these authors to take account of these abrupt variations in the stochastic processes that reflect the crude oil price growth. The unit root tests with and without breaks used by Postali et al., (2006) for the oil price series, rejects the unit root for long samples. The first step to carry out the present research is the examination of the stationary nature of the variables. The unit root tests without structural break, which are considered as non-stationary tests, include Augmented Dickey and Fuller (1979) and Philips and Perron (1988). The unit root tests are applied in the following chapter. If we reject the stationary hypothesis, the rejection may be due to the presence of at least one significant break in the series. In that case, unit root test is done with a structural break in section 2. Unit root test with an unclear structural break (endogenous) includes Perron (1997) test. If these tests do not permit to identify the appropriate processes, the choice is based on the simulation of continuous-time stochastic processes and the mean square error between the simulated prices and the market ones. The simulation requires the parameter estimation of continuous-time stochastic processes. In section 3, we estimate the parameters of usual continuous-time stochastic processes by method of the Maximum Likelihood Estimation (hereafter, MLE). The simulation of continuous-time stochastic processes and the Monte-Carlo simulation of the mean square error (hereafter MSE) between the simulated prices and the market ones are presented in section 4.

2. Unit Root Tests with and Without Breaks

2.1 Unit root tests without breaks

The first step to achieve to the main purpose of this research is the examination of the stationary nature of data. We examine the mean reversion of the crude oil price. Econometric analysis is performed by using unit-root tests: (1) Augmented Dickey and Fuller (1979) test (hereafter, ADF) and Phillips and Perron (1988) test (hereafter, PP) whose null hypotheses are non-stationary. The price of each barrel of the Iranian light and heavy crude oil is used in dollars on a monthly basis for the period from Jan 1980 to Aug 2009 (Figure 1). They are obtained from U.S energy information administration. The results of unit root tests without breaks are reported in tables 1 and 2.

Figure 1. The price of each barrel of the Russian Ural crude oil in dollars

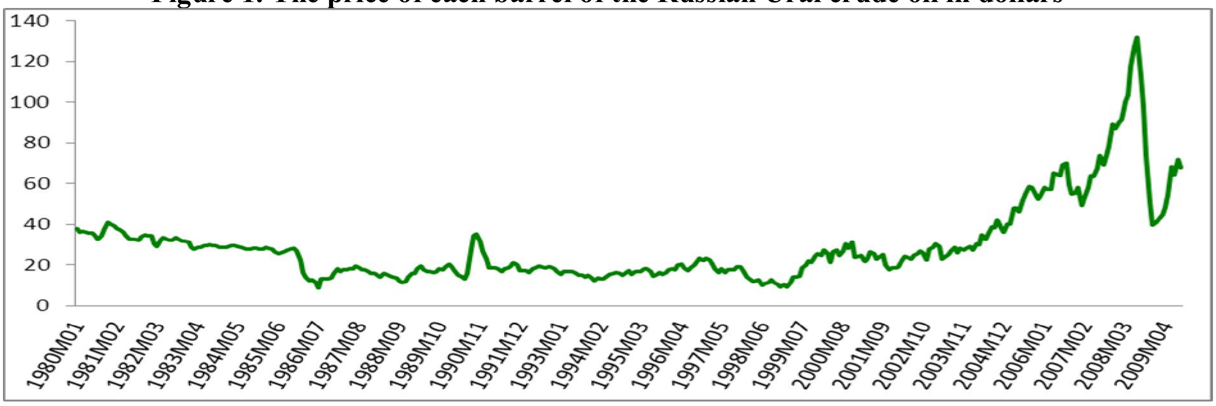


Table 1. Results of ADF test for the crude oil price series

Type of test	Optimal break	The test statistic	Critical value at risk level %1	Critical value at risk level %5	Critical value at risk level %10	Probability Value (P value)
ADF with intercept	5	-1.59	-3.44	-2.86	-2.57	0.48
ADF with Intercept And time trend	5	-2.32	-3.98	-3.42	-3.13	0.41

Table 2. Results of PP test in for the crude oil price series

Type of test	Bandwidth	The test statistic	Critical value at risk level %1	Critical value at risk level %5	Critical value at risk level %10	Probability Value (P value)
PP with intercept	4	-2.03	-3.44	-2.86	-2.57	0.27
PP with Intercept And time trend	4	-2.73	-3.98	-3.42	-3.13	0.22

As can be seen in above tables, for ADF test with intercept and without the time trend the computed test statistic is greater than the critical values at the 1%, 5% and 10% risk levels, so we accept the unit root null hypothesis. This result repeats in presence of the time trend. For PP test, the null hypothesis based on the non-stationary of series under investigation, is accepted in different states.

2.2 Unit root test with a structural break

Perron (1989) indicated that unit root tests presented above are biased toward the non-stationary hypothesis when series present a structural break. In fact, a result in favor of a random-walk could be stationary if dummy variables that present a break in the intercept and/ or in the slop of the trend function are contained of the regression of the ADF test. Findings of Perron (1989) indicate that the majority of macroeconomic time series are stationary around a broken trend. The Perron (1997) test, which allows for a one-time break, is used to the crude oil price series.

2.2.1. The Perron (1997) unit root test with an endogenous structural break

Perron (1997) proposed two estimation methods. The first, referred to as Additive Outliers (hereafter, AO), allows for an abrupt change. The second, referred to as Innovational Outliers (hereafter, IO), allows for a gradual change. Perron can distinct two patterns for Innovational Outliers of each other. (IO1): for gradually change in intercept and (IO2): the pattern that let the application of the gradual changes regarding in both cases the intercept and the slope of the trend function. In this research, regarding the behavior of the described series, the IO2 and AO results are mentioned.

Table 3. Results of PP test with an endogenous structural break for the crude oil price series

Type of test	Optimal break	The test statistic	Critical value at risk level %1	Critical value at risk level %5	Critical value at risk level %10	Break dates
IO2	11	- 4.90	-5.57	-4.91	-4.59	2000:10
AO	11	- 4.75	- 4.87	- 4.34	-4.04	1999:10

The results of table 3 shows that in a high risk interval the stationary hypothesis is accepted, so we cannot conclude in favor of a mean-reverting process for the oil price model for sure. In order to choose the appropriate stochastic process for this series, we simulate different continuous-time stochastic processes and the mean square error between the simulated prices and the market ones in section 4.

3. Estimation of the Crude Oil Price Process

3.1. Maximum likelihood estimating procedure

The MLE consists in maximizing the density function. The density functions of the Geometric Brownian Motion and Ornstein-Uhlenbeck (hereafter OU) process are respectively:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi\Delta t}} Exp\left[-\frac{\left[\log(x/x_{\circ}) - (k-(1/2)\sigma^2)\Delta t\right]^2}{2\Delta t\,\sigma^2}\right] \tag{1}$$

$$p(x)=\frac{1}{\sqrt{\pi\frac{\sigma^2}{k}\left\{1-Exp\left[-2k(t-t_\circ)\right]\right\}}}Exp\left[-\frac{\left\{(x-a)-(x_\circ-a)Exp\left[-k(t-t_\circ)\right]\right\}^2}{\frac{\sigma^2}{k}\left\{\left[1-Exp-2k(t-t_\circ)\right]\right\}}\right] \tag{2}$$

Table 4. Parameter estimates

Geometric Brownian Motion $dX_t = kX_t dt + \sigma X_t d\beta_t$			
Parameters	K	σ	
Values	0.03985	0.16524	
Maximum log – likelihood	1.78		
Ornstein-Uhlenbeck Process $dX_t = k(a-X_t)dt + \sigma d\beta_t$			
Parameters	a	K	σ
Values	0.1617	0.0096	0.298
Maximum log – likelihood	1.47		

We estimate the parameters by maximizing the log-likelihood function.[1] The estimation results are reported in Table 4.

4. Simulation of the Crude Oil Price Process

In this section, we simulate the stochastic processes, fitted in the prior section, and compute the mean square error between the simulated prices and the market ones.

4.1. Geometric Brownian motion: price and mean square error simulation

Process is:

$$dp_t = kp_t dt + \sigma p_t dZ_t \tag{3}$$

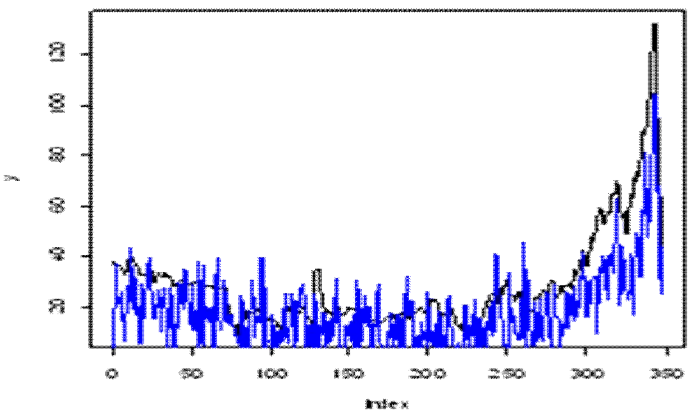
Its discreet-time form is:

$$P_t = P_{t-1}Exp\left[\left(k-\frac{\sigma^2}{2}\right)\Delta t+\sigma N(0,1)\sqrt{\Delta t}\right] \tag{4}$$

The sample path simulated by the Geometric Brownian Motion (GBM), follows the historical crude oil price curve (Figure 3).

Figure 3. Simulation of the GBM

Market Prices Simulated Prices



3- Mathematic program is applied for maximize the density functions.

For comparison, we also compute the mean square error:

$$MSE = \frac{\sum_{i=1}^{N} (p_i^s - p_i)^2}{N} \qquad (5)$$

Where, MSE is the mean squared error, p_i^s is the simulated prices, p_i is the market prices and N is the number of observations. To reach a high level of accuracy, a large number of simulations Monte-Carlo procedure is carried out as follow. The number of repetition is 1000.

4.2. Ornstein-Uhlenbeck process: price and mean square error simulation

Process is:

$$dX_t = k(a - X_t)dt + \sigma dZ_t \qquad (6)$$

Its discreet-time form is:

$$X_t = X_{t-1}e^{-k\Delta t} + a(1 - e^{-k\Delta t}) + \sigma\sqrt{\frac{1 - e^{-2k\Delta t}}{2k}}N(0,1) \qquad (7)$$

Note that this process can assume even negative values. So in our case (commodity prices),
We simulate:

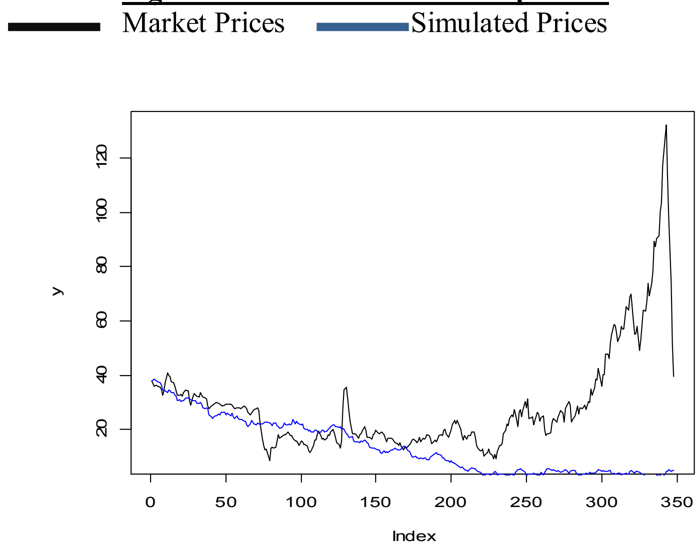
$$p_t = Exp\left[X_t - 0.5V(X_t)\right] \qquad (9)$$

With

$$V(X_t) = \frac{\sigma^2}{2k}(1 - e^{-2k\Delta t}) \qquad (10)$$

The sample paths simulated by the OU process shows that this process cannot describe the market price evolution (Figure 4).

Figure 4. Simulation of the OU process



We note an increase in the mean square error of the OU process in contrast with the GBM (Table 5). The simulated sample paths compared with the historical oil price curve, and the mean square error frequencies between the simulated prices and the market ones show that: (1) The OU process cannot characterize the crude oil price evaluation. (2) The mean square error value of the OU process is more than the Geometric Brownian Motion (GBM), therefore The GBM is the best stochastic process for the crude oil price.

Table 5. The mean square error values

Stochastic Models	MSE
Geometric Brownian Motion	275/8329
Ornstein-Uhlenbeck Process	873/708

5. Conclusion

The goal of this paper is to determine the best continuous-time stochastic processes for the crude oil price. The price of each barrel of the Iranian light and heavy crude oil is used in dollars on a monthly basis for the period from Jan 1980 to Aug 2009 (Figure1). They are obtained from U.S energy information administration. The methodology followed: (1) Running the unit root tests with and without breaks. (2) Simulating continuous-time stochastic processes and computing the mean square error between the simulated prices and the market ones. The simulation shows that the Geometric Brownian Motion is the best stochastic process to fit the Russian crude oil price.

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