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Do Long-memory GARCH-type-Value-at-Risk Models Outperform None-and Semi-parametric Value-at-Risk Models?

Onder Buberkoku*

Department of Finance, Faculty of Business Administration, Yuzuncu Yil University, Turkey. *Email: onderbuber@gmail.com

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ABSTRACT

As a result of the 2007-2008 global financial crisis, traditional value-at-risk (VaR) models used to measure the market risk have been criticised for their inaccuracy. Therefore, alternative models such as long-memory GARCH-type based VaR models have been receiving increased attention in recent literature. In this regard, this study compares the one-day-ahead out-of-sample VaR forecasting performances of FIGARCH, HYGARCH, and FIAPARCH models under normal, student t, and skewed student t distribution assumptions with FHS and HS model performances, which are the most commonly applied models especially by commercial banks in practice, for eight different financial variables including energy commodities (West intermediate crude oil and New York Harbour Conventional Gasoline regular (NYHCGR)), stock indices (NIKKEI 225 stock market index and TSEC weighted stock index), foreign exchange rates (Euro/US Dollar (EUR/USD) and Japanese Yen/USD (JPY/USD)), and precious metals (gold and copper). Results clearly show that the FHS model is the most appropriate model for long trading positions, to which the relevant literature has paid more attention, whereas for short trading positions the HYGARCH model under skewed student t distribution assumption should be preferred.

Keywords: Long-memory GARCH-type Models, Value-at-risk, Historical Simulation, Filtered Historical Simulation
JEL Classifications: C58, G15, G17, Q02

1. INTRODUCTION

Value-at-risk (VaR) is the major tool used to measure the market risk of a portfolio. Among alternative VaR models, generalised autoregressive conditional heteroskedasticity (GARCH)-based VaR models are commonly used to measure the market risk of financial variables. However, especially after the 2007-2008 global financial crisis period, such traditional models have been criticised because of their inability to meet accurately the market losses. This issue is mainly attributed to the fact that standard GARCH-type VaR models suffer from a number of shortcomings, such as the use of short sample sizes and normal distribution assumptions. Additionally, these kinds of models also assume that the volatility of financial assets exhibits short-memory property. In contrast, the relevant literature reports that the volatility of many financial assets exhibits long memory property (Beine et al., 2002; Baillie et al., 2007; Wu and Shieh, 2007; Kang et al., 2009; Arouri et al., 2012; Chkili et al., 2014; Bentes, 2015). Therefore, extant studies have paid increasing attention to alternative VaR models in order to measure the market risk more accurately. In this regard, long-memory GARCH-type models such as the fractionally integrated GARCH (FIGARCH) model, introduced by Baillie et al. (1996), the hyperbolic GARCH (HYGARCH) model, proposed by Davidson (2004), and the fractionally integrated asymmetric power ARCH (FIAPARCH) model, developed by Tse (1998), have gained significant attention. For example, Lanouar (2016) forecasts the volatility of West intermediate crude oil (WTI), heating oil, propane, and RBOB regular gasoline future prices based on the alternative models, and indicates that FIGARCH and fractionally integrated exponential GARCH (FIEGARCH) models under student t distribution assumption perform better than GARCH, exponential GARCH (EGARCH), and Markovswitching GARCH models. Kang et al. (2009) find that in most cases, the FIGARCH model provides superior performance for Brent, WTI, and Dubai crude oil markets. Aloui and Mabrouk (2010) test the performance of alternative long-memory GARCH models for major crude oil and gas commodities and show that the FIAPARCH model with skewed student t distribution outperforms the other models. Chkili et al. (2014) use a broad set of the most popular linear and non-linear GARCH-type models for crude oil, natural gas, gold, and silver commodities, and find that the FIAPARCH model with student t distribution is the most accurate. Mabrouk and Aloui (2010), Aloui and Hamida (2014; 2015), Mabrouk and Saadi (2012), and Degiannakis (2004) indicate that FIAPARCH with skewed student t distribution produces the most accurate results for stock indices. Chkili et al. (2012) examine the conditional volatility dynamics of stock returns and exchange rates and indicate that both the univariate FIAPARCH and bivariate constant conditional correlation (CCC)-FIAPARCH models are much more appropriate than standard GARCH-type model specifications in nearly all cases. Beine et al. (2002) indicate that the exchange rate volatility measure of the FIGARCH model outperforms the GARCH one. Wu and Shieh (2007) estimate that different GARCH-type VaR models for T-Bonds interest rate show that long-memory GARCH-type models perform better than standard short-memory GARCH-type models. Bentes (2015) employs the GARCH, integrated GARCH (IGARCH), and FIGARCH specifications to investigate volatility behaviour of gold returns and concludes that FIGARCH is the best model to forecast their volatility. Arouri et al. (2012) reveal that the autoregressive fractionally integrated moving average (ARFIMA)-FIGARCH model outperforms other several popular volatility models for four major precious metal commodities (gold, silver, platinum, and palladium). Similarly, Demiralay and Ulusoy (2014) find that the FIAPARCH model with student t distribution provides better forecast accuracy for the same four major precious metal commodities. Lastly, Baillie et al. (2007) show that FIGARCH models are superior to standard GARCH models for six different commodities (corn, soybeans, cattle, hogs, gasoline, and gold).

It can be seen from these and similar studies that long-memory GARCH-type models have emerged as a generally better choice than short-memory GARCH-type models, due to the fact that these models capture the stylised facts of financial time series more accurately. However, an alternative approach to parametric models is to use historical simulation and/or filtered historical simulation models, which are popular models both in literature and practice. These models are commonly used and employed in a wide range of financial assets in the relevant literature, such as stock indices, exchange rates, interest rates, energy commodities, derivative securities, precious metals, and even electricity markets (e.g. Hendricks, 1996; Cabedo ve Moya, 2003; Gençay ve Selçuk, 2004; Vlaar, 2000; Barone-Adesi et al., 2002; Chan and Gray, 2006; Marimoutou et al., 2009; Hammoudeh et al., 2011; Dario and Stefano, 2012; Hammoudeh et al., 2013). The popularity of these models is because they take into account the stylised facts of financial return series, such as skewness, excess kurtosis, and non-normal distribution. Additionally, unlike the GARCH-based VaR models, FHS and HS models do not need a pre-specified distribution assumption and can also be used to measure the market risk of non-linear positions.

In this regard, the aim of this study is to compare the VaR performances of long-memory GARCH-type models with HS and FHS models for eight financial variables: WTI, NYHCGR, EUR/US, JPY/USD, NIKKEI 225 stock market index, TSEC weighted stock index, copper, and gold. This study's main contribution to the literature is that it examines whether or not the promising results provided by long-memory GARCH-type models are also valid when their performances are compared with two other important models: FHS and HS. One of the main drawbacks in the relevant literature is that, in most cases, long-memory GARCH-type based studies compare the out-of-sample VaR forecasting performance of FIGARCH, FIAPARCH, and HYGARCH models with short-memory GARCH-type models' performances. In other words, they have not paid enough attention to compare the out-of-sample VaR forecasting performance of these long-memory GARCH-type VaR models with a non-parametric (i.e., HS) and/or semi-parametric (i.e., FHS) model, as of yet. Additionally, both long and short trading positions are taken into account, and expected shortfall (ES), which is pointed out by Giot and Laurent (2003) amongst others, is another important part of the risk management process since it sheds lights on how much a risk manager can lose on average when the relevant VaR model fails. Furthermore, it is also calculated in each individual case. These points are also considered as contributions to the relevant literature due to the fact that although there are many studies calculating ES by taking into account different trading positions (i.e., short and long trading positions based on parametric VaR models), it is observed that for FHS and HS models in particular, more studies are needed in order to report on how the two models perform for short trading positions and what ES values they produce for alternative trading positions.

The rest of the paper is organised as follows: Section 2 shows data and methodology, Section 3 provides empirical results, and Section 4 presents the concluding remarks.

2. DATA AND METHODOLOGY

2.1. Data

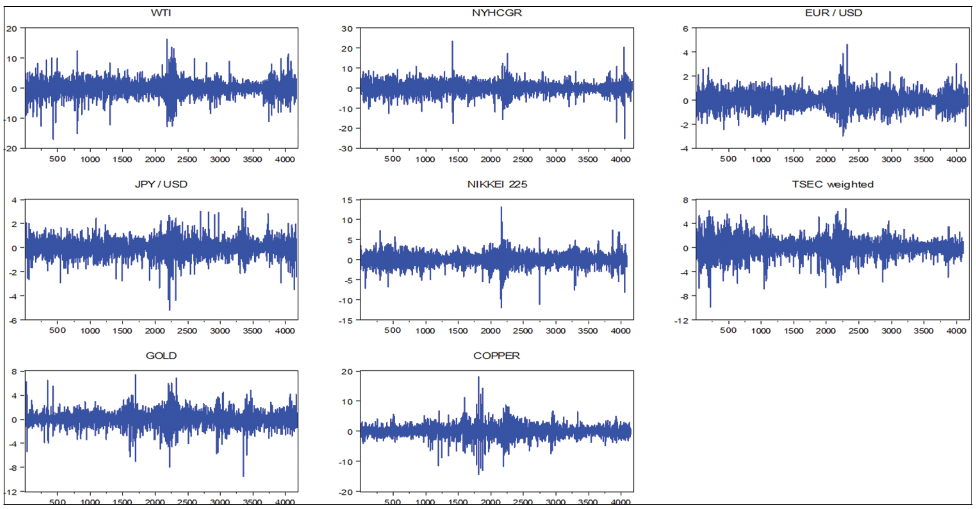
The study uses the daily closing spot prices of four important asset classes, including energy commodities (WTI and NYHCGR), stock indices (NIKKEI 225 stock market index and TSEC weighted stock index), foreign exchange rates (EUR/USD and JPY/USD), and precious metals (gold and copper). Detailed information about the data is presented in Table 1. The data for each asset class covers the period from January 04, 2000 to August 04, 2016, consisting of nearly 4200 observations for each financial variable. Following the studies using long-memory GARCH-type models, the data set is divided into two subperiods, and the last 1000 observations are left out of our sample analysis. The continuously compounded daily returns (r_t) of each financial variable are calculated as follows:

Table 1: Data explanations and sources

Financial variable	Description	Data source
WTI	In US dollar per barrel	US Energy Information Administration
NYHCGR	In US dollar per barrel	US Energy Information Administration
NIKKEI 225	Japan stock index, in local currency	Finance yahoo
TSEC weighted index	Taiwan stock index, in local currency	Finance yahoo
EUR/USD	Euro into US dollar	Bank of England
JPY/USD	Japanese yen into US dollar	Bank of England
Gold	London fixings, London bullion Market association in US dollar per troy ounce	Bank of England
Copper	London metal exchange, in US dollar per tonne	Quandl

WTI: West intermediate crude oil, NYHCGR: Harbour Conventional Gasoline regular, NIKKEI 225: Japan stock index, TSEC weighted index: Taiwan stock index, EUR/USD: Euro into US dollar, JPY/USD: Japanese yen into US dollar

Figure 1: Plots of the return series



$$r_t=100*[\ln(P_t)-\ln(P_{t-1})] \quad (1)$$

Where P_t is the closing price on day t.

The graphs of the return series are presented in Figure 1.

2.2. Methodology

2.2.1. Long-memory GARCH-type VaR models (parametric VaR models)

In this study, FIGARCH, HYGARCH, and FIAPARCH models are used as long-memory GARCH-type models. The FIGARCH (1, d, 1) model is given by:

$$r_t=\mu+\varepsilon_t,\ \varepsilon_t=\sigma_t\epsilon_t,\ \epsilon_t\sim(0,1) \quad (2)$$

$$h_t=\omega_0+\beta h_{t-1}+\left[1-(1-\beta L)^{-1}(1-\phi L)(1-L)^d\right]\varepsilon_t^2 \quad (3)$$

Where $\omega_0>0, \beta<1, \phi<1, 0\leq d\leq 1$, L is the lag operator, and d is the fractional integrator parameter. Equation (2) and equation (3) show the conditional mean and variance equations, respectively.

The HYGARCH (*1, d, 1*) model can be defined as follows:

$$h_t=\omega_0+\left[1-(1-\beta L)^{-1}\phi L(1+\alpha((1-L)^d-1)\right]\varepsilon_t^2 \quad (4)$$

However, the FIGARCH and HYGARCH models do not consider the asymmetry in volatility. Therefore, the FIAPARCH model, which covers both the long-memory and asymmetry in conditional variance, is also employed. The FIAPARCH (1, d, 1) model is written as:

$$h_t^{\delta}=\omega_0(1-\beta L)^{-1}+\left[1-(1-\beta L)^{-1}(1-\phi L)(1-L)^d\right](|\varepsilon_t|-\gamma\varepsilon_t)^d \quad (5)$$

Where $\omega_0>0$, $\delta>0$, $\phi<1$, $\beta\leq 1$, $-1<\gamma<1$ γ is the leverage coefficient, and δ is the power term parameter.

However, long-memory GARCH-type VaR models need a pre-specified distribution assumption. It is not an easy task to decide which distribution assumption should be preferred to get more accurate forecasting results. Therefore, in most analyses, two or three distribution assumptions are used together. Similarly, following the relevant literature, in this study the FIGARCH, HYGARCH, and FIAPARCH models are estimated under Gaussian normal, the student t, and the skewed student t distribution assumptions.

Assuming a standard normal distribution, VaR is given by:

$$VaR_{t,long\ position}^{downside\ market\ risk}=\mu_t-z_{\alpha}\sqrt{h_t} \quad (6)$$

$$VaR_{t,short\ position}^{upside\ market\ risk}=\mu_t-z_{1-\alpha}\sqrt{h_t} \quad (7)$$

Where μ_t is the mean conditional return and $\sqrt{h_t}$ is the conditional standard deviation, both of which are obtained from the relevant long-memory GARCH-type models. Further, z_α is the left *ath*-quantile and $z_{1-\alpha}$ is the right (1 - α)*th* quantile of the standard normal distribution, respectively.

Under the standard normal distribution assumption, the parameters of FIGARCH, HYGARCH, and FIAPARCH models are estimated using the log-likelihood function of Gaussian normal distribution ($LogLL_{normal}$), which is given by:

$$LogLL_{normal} = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi) + \ln(\sigma_t^2) + z_t^2\right] \tag{8}$$

Where, σ_t^2 is the variance, $z_t = \varepsilon_t / \sigma_t$, and T is the number of observations.

Assuming a student t distribution, the VaR is given by:

$$VaR_{t,long\ position}^{downside\ market\ risk} = \mu_t - student\ t_{\alpha,v}\sqrt{h_t} \tag{9}$$

$$VaR_{t,short\ position}^{upside\ market\ risk} = \mu_t - student\ t_{1-\alpha,v}\sqrt{h_t} \tag{10}$$

Where *student* $t_{\alpha,v}$ and *student* $t_{1-\alpha,v}$ are the left and right quantiles of the student t distribution with the v degrees of freedom.

The log-likelihood function of student t distribution (*LogLLstudent t*) is written as follows:

$$LogLL_{student\ t} = T\left\{\ln\Gamma\left(\frac{v+1}{2}\right) - \ln\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\ln\left[\pi(v-2)\right]\right\} - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+v)\left[\ln(1+\frac{z_t^2}{\sigma_t^2(v-2)})\right]\right] \tag{11}$$

Where v is the number of degrees of freedom with $v > 2$, and Γ (.) is the gamma function.

Assuming the skewed student t distribution, VaR is given by:

$$VaR_{t,long\ position}^{downside\ market\ risk} = \mu_t - skwstudent\ t_{\alpha,v,\xi}\sqrt{h_t} \tag{12}$$

$$VaR_{t,short\ position}^{upside\ market\ risk} = \mu_t - skwstudent\ t_{1-\alpha,v,\xi}\sqrt{h_t} \tag{13}$$

Where *skwstudent* $t_{\alpha,v,\xi}$ and *skwstudent* $t_{1-\alpha,v,\xi}$ are the left and right quantiles of skewed student t distribution with the v degrees of freedom.

The log-likelihood function of skewed student t distribution (*LogLLstudent t*) can be defined as follows:

$$LogLL_{skwstudent\ t} = T\left\{\begin{array}{l}\ln\Gamma\left(\frac{v+1}{2}\right) - \ln\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\ln\left[\pi nn\Gamma\ )\right] + \\ \ln\left(\frac{2}{k+\left(1/k\right)}\right) + \ln(s)\end{array}\right\} - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+v)\ln\left[1+\left(1+\frac{(sz_t+m)^2}{v-2}\right)k^{-2I_t}\right]\right] \tag{14}$$

Where k is the asymmetry parameter, and the value of ln (k) determines the degree of the asymmetry in the distribution of the relevant financial return series.

2.2.2. Historical simulation (non-parametric VaR model)

Parametric VaR models have some disadvantages. For example, they need a distribution assumption (a theoretical distribution) and a model to estimate the time-varying conditional volatilities. Unfortunately, as commonly reported in the relevant literature, unsuitable distribution assumptions and model selections may result in large biases. An alternative approach to parametric VaR models is to use the HS model, which is one of the most commonly used models by financial institutions because of its simplicity. Generally, its simplicity arises from the fact that the HS model directly uses the empirical distributions of the actual returns and assumes that the empirical distribution of actual past returns is stable over time. Therefore, the empirical distribution of past returns can be used to predict expected future losses (Marimoutou et al., 2009; Toggins, 2008; Abad et al., 2014).

Under this framework, the HS model can be defined as:

$$VaR_{t,\alpha\ long\ position}^{downside\ market\ risk} = quantile\left\{\{r_t\}_{t=1}^{T}\right\} \tag{15}$$

$$VaR_{t,1-\alpha\ short\ position}^{upside\ market\ risk} = quantile\left\{\{r_t\}_{t=1}^{T}\right\} \tag{16}$$

Where r_t is the logarithmic historical return, α is the left *ath*-quantile, and 1-α is the right (1-α)*th*-quantile of the relevant empirical distribution.

However, the HS model also has some disadvantages. For example, if the sample size is not long enough to capture the extreme events that occurred in the past, it may underestimate the actual market risk. Besides, the HS model does not consider the volatility clustering and time-varying volatility characteristics of financial return series, so it does not consider the fact that the risk of the relevant financial variable can change over time (Barone-Adesi et al., 2002; Degiannakis et al., 2013; Toggins, 2008; Abad et al., 2014)

2.2.3. Filtered historical simulation (semi-parametric VaR model)

The FHS model was developed by Hull and White (1998) and Barone-Adesi et al. (1999). The main contribution of the FHS model is that it adjusts the historical returns in order to reflect the

Table 2: Summary statistics, unit root, and long memory tests for return series

	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
Panel A: Summary statistics (%)								
Mean	0.011876	0.017197	0.001847	−0.0005	−0.003823	0.000736	0.022982	0.037622
Median	0.072815	0.102059	0.000002	0.00002	0.029493	0.041690	0.005473	0.018550
Maximum	16.41370	23.50513	4.620792	3.342812	13.23459	6.524616	18.16926	7.417305
Minimum	−17.09179	−25.33428	−3.003101	−5.215648	−12.11102	−9.936013	−14.49966	−9.596165
Standard deviation	2.509521	2.814234	0.639658	0.650665	1.560554	1.417850	1.877527	1.160514
Skewness	−0.148465*	−0.084253*	0.102371*	−0.335093*	−0.390440*	−0.243954	−0.091477*	−0.266178*
Kurtosis	4 261985*	6.205303*	2.122719*	4.052216*	5.972392*	3.161976*	8.23669*	5.694289*
Jarque-Bera	3168.4* (0.000)	6688.9* (0.000)	790.00* (0.000)	2930.4* (0.000)	6176.5* (0.000)	1747.4* (0.000)	11731.4* (0.000)	5713.1* (0.000)
Panel B: ARCH effects tests								
Q2 (12)	1335.5* (0.000)	945.92* (0.000)	489.67* (0.000)	279.5* (0.000)	3160.2* (0.000)	1436.4* (0.000)	877.76* (0.000)	529.79* (0.000)
ARCH (12)	47.725* (0.000)	60.901* (0.000)	24.146* (0.000)	16.704* (0.000)	104.43* (0.000)	49.414* (0.000)	56.108* (0.000)	22.822* (0.000)
Panel C: Unit roots tests								
ADF	−66.65* (0.000)	−64.02* (0.000)	−64.05* (0.000)	−66.05* (0.000)	−66.22* (0.000)	−61.07* (0.000)	−72.545* (0.000)	−65.98* (0.000)
PP	−66.7* (0.000)	−64.03* (0.000)	−64.06* (0.000)	−66.06* (0.000)	−66.38* (0.000)	−61.05* (0.000)	−72.290* (0.000)	−65.99* (0.000)
KPSS	0.196763	0.131547	0.202673	0.117614	0.229846	0.123717	0.296003	0.165884
Panel D: Lo's (1991) R/S test								
Return								
q=1	1.2183	0.9765	1.5094	1.3604	1.1677	1.1722	1.4529	1.3654
q=2	1.2353	0.9776	1.5158	1.3718	1.1768	1.1535	1.4879	1.3804
q=5	1.2381	1.0015	1.5124	1.3894	1.2132	1.1453	1.5095	1.3773
Squared return								
q=1	4.0360*	3.7793*	4.1678*	3.3299*	3.5794*	5.7415*	5.3347*	4.4805*
q=2	3.7554*	3.5285*	3.9726*	3.1693*	3.1854*	5.3248*	5.0022*	4.2700*
q=5	3.1631*	2.9786*	3.5750*	2.9413*	2.5523*	4.4767*	4.4602*	3.8192*
Absolute return								
q=1	5.7722*	6.2618*	4.2328*	3.9830*	3.8386*	6.9232*	7.9679*	6.0851*
q=2	5.3246*	5.8670*	4.0799*	3.8036*	3.5143*	6.3842*	8.8554*	5.7545*
q=5	4.4367*	5.0091*	3.6478*	3.4796*	2.8721*	5.2816*	6.2170*	5.0220*

q denotes the lag parameters used for Lo's (1991) R/S test. * denotes the 5% significance level. Figures in parentheses are the probability values. Lo's (1991) R/S test has the null hypothesis of no long memory. Kurtosis refers to excess kurtosis. The unit root tests are estimated in form with constant only

Table 3: FIGARCH model estimation results

Model	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
FIGARCH model with standard normal distribution assumption								
μ (Mean)	0.05309**	0.04034	0.00562	0.00676	0.04856*	0.04751*	0.00231	0.04125*
ω_0 (Variance)	0.13900**	0.36717*	0.00104	0.01486*	0.06189*	0.05592**	0.17533**	0.07018*
d (Long memory)	0.42192*	0.35368*	0.87608*	0.36277*	0.61667*	0.39653*	0.31433*	0.35421*
φ_1 (ARCH)	0.39326*	0.18549*	0.05812	0.47086*	0.10418**	0.08947	0.20688	0.29444*
β_1 (GARCH)	0.67805*	0.43486*	0.94161*	0.73940*	0.64281*	0.44821*	0.42537*	0.55702*
LL	−9245.016	−9815.679	−3790.758	−3950.07	−7119.94	−6614.48	−7899.735	−6213.65
AIC	4.440718	4.714680	1.820944	1.897370	3.487489	3.231378	3.811348	2.966914
Q^2 (20)	16.48	0 22.67	0 7.94	20.2	25.18	0 31.5*	23.5	8.79
ARCH (20)	0.82298	1.1695	0.40366	0.9975	1.2452	1.4906**	1.1895	0.44674
RBD (20)	16.4535	−7.30269	6.20187	91.9086*	20.2146	29.7734**	−3.13344	8.23419
FIGARCH model with student t distribution assumption								
μ (Mean)	0.06735*	0.06877*	0.00702	0.00956	0.06211*	0.06368*	0.01522	0.04059*
ω_0 (Variance)	0.09020*	0.34926*	0.00090**	0.01884*	0.06528*	0.02997*	0.12319*	0.04982*
d (Long memory)	0.46179*	0.33175*	0.88916*	0.35007*	0.50915*	0.47219*	0.33615*	0.50249*
φ_1 (ARCH)	0.37137*	0.21396*	0.04382	0.38408*	0.07611	0.15076*	0.32450*	0.21025*
β_1 (GARCH)	0.71818*	0.46331*	0.94074*	0.67763*	0.55263*	0.60984*	0.58958*	0.68049*
v (Tail)	7.301945*	8.160908*	9.335456*	5.60065*	8.88828*	6.90482*	5.82089*	4.20995*
LL	−9145.989	−9754.760	−3751.075	−3804.482	−7068.99	−6541.257	−7776.89	−5985.625
AIC	4.393658	4.685914	1.802387	1.828008	3.463041	3.196123	3.752601	2.858600
Q^2 (20)	20.74	33.4*	8.46	19.53	48.8*	41.3*	32.6*	0 20.4
ARCH (20)	1.0451	1.7029*	0.42707	0.96975	2.4586*	1.8929*	1.6067*	0.98663
RBD (20)	17.7914	−76.8742	9.38926	24.3533	12.7103	15.0623	15.8009	−31.7103
FIGARCH model with skewed student t distribution assumption								
μ (Mean)	0.04709	0.04805	0.00415	0.00402	0.04477*	0.04452*	0.00590	0.03724*
ω_0 (Variance)	0.08748*	0.34756*	0.00089**	0.01896*	0.06078*	0.02743**	0.12084*	0.04994*
d (Long memory)	0.46179*	0.32926*	0.88719*	0.34807*	0.50497*	0.46026*	0.33891*	0.50217*
φ_1 (ARCH)	0.36760*	0.21367*	0.04484	0.38605*	0.08220	0.15030*	0.32621*	0.20986*
β_1 (GARCH)	0.71607*	0.46030*	0.94061*	0.67676*	0.54998*	0.59746*	0.59345*	0.68009*
v (Tail)	7.433535*	8.252045*	9.38432*	5.63525*	9.75957*	7.45199*	5.83322*	4.21305*
ζ (Asymmetry)	−0.05605*	−0.05614*	−0.02818	−0.03469**	−0.08686*	−0.08752*	−0.02442	−0.08945
LL	−9142.718	−9751.380	−3750.179	−3803.14	−7061.38	−6532.69	−7776.206	−5985.518
AIC	4.392567	4.684772	1.802436	1.827842	3.459806	3.192430	3.752751	2.859026
Q^2 (20)	20.93	33.25*	8.39	19.29	45.2*	41.3*	32.5*	20.45
ARCH (20)	1.0525	1.6968*	0.42387	0.95769	2.2720*	1.8900*	1.5988*	0.98925
RBD (20)	19.5468	−54.7680	9.07197	22.5033	12.9171	14.4557	15.2828	−30.9102

*and ** denote the 5% and 10% significance levels, respectively. LL is the value of maximised log likelihood. AIC is the Akaike (1974) information criterion. For normal distribution assumption, robust standard errors are estimated with sandwich formula

changing market condition. In other words, it uses a methodology that is based on combining the main characteristics of GARCH-type models and the HS model. More specifically, it first estimates the conditional standard deviations by using GARCH-type models to generate standardised returns that are considered to be more appropriate for simulation analysis compared to raw historical returns used in HS analysis. Then, like the HS model, the quantile of standardised returns is used for VaR calculation (Marimoutou et al., 2009; Abad et al., 2014; Louzis et al., 2004; Toggins, 2008) With such an approach, the FHS model is able to take into account stylised facts of financial return series such as volatility clustering, skewness, excess kurtosis, and non-normal distribution (Angelidis et al., 2007).

In this framework, the FHS model can be calculated as:

$$VaR_{t,\alpha\ long\ position}^{downside\ market\ risk} = \mu_t + \sqrt{h_t}\, quantile\left\{\{z_t\}_{t=1}^{T}\right\} \quad (17)$$

$$VaR_{t,1-\alpha\ short\ position}^{upside\ market\ risk} = \mu_t + \sqrt{h_t}\, quantile\left\{\{z_t\}_{t=1}^{T}\right\} \quad (18)$$

Where z_t is the standardised historical return[1], $\sqrt{h_t}$ is the most recent estimate of the conditional standard deviation by standard GARCH model, α is the left *ath*-quantile, and *1-α* is the right (1-*α*) *th*-quantile of the relevant empirical distribution.

2.2.4. Backtesting procedure

VaR models are only meaningful tools as far as they forecast future potential losses accurately (Jorion, 2007). Therefore, evaluating the VaR models' accuracy, called the backtesting procedure, is another crucial part of the financial risk management process. In the VaR calculations of this study, quantiles ranging from 0.95 to 0.9975 are used for long trading positions, and those from 0.05 to 0.0025 are used for short trading positions. Then, in order to determine the most accurate VaR model, the forecasted one-day-ahead VaR forecasts are

1 Following the genearal approach in the relavant literaure, the standard GARCH model under normal distribution assumption is used as a filter for the FHS model.

Table 4: HYGARCH model estimation results

Model	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
HYGARCH model with standard normal distribution assumption								
μ (Mean)	0.05299*	0.03698	0.00562	0.00705	0.04760*	0.04736*	0.00260	0.04089*
ω_0 (Variance)	0.11654	0.15585	0.00167	0.00836	0.08547*	0.04867	0.13879	0.09072*
d (Long memory)	0.39796*	0.24405*	0.89898*	0.29952*	0.76770*	0.37156*	0.25359*	0.46949*
φ_1 (ARCH)	0.40284*	0.14673	0.04099	0.50999*	0.05431	0.07588	0.15921	0.26879*
β_1 (GARCH)	0.67056*	0.32245*	0.94488*	0.73907*	0.72716*	0.41646**	0.33979	0.61738*
Log ($\tilde{\alpha}$) (Hyperbolic)	0.01834	0.14947	−0.00348	0.07515	−0.03006	0.01890	0.08973	−0.07175
LL	−9244.85	−9813.55	−3789.73	−3949.46	−7118.12	−6614.332	−7898.68	−6212.143
AIC	0 4.441118	4.714141	1.820932	1.897558	3.487082	3.231795	3.811322	2.96671
Q^2 (20)	16.24	20.80	8.37	0 18.15	24.65	31.3*	22.9	0 9.05
φ_1ARCH (20)	0.80767	1.0866	0.42568	0.89806	1.2300	1.4865**	1.1616	0.45998
RBD (20)	16.8318	13.4096	8.927	48.1994*	4.96197	29.8417**	0.13376	7.64968
HYGARCH model with student t distribution assumption								
μ (Mean)	0.06712*	0.06643**	0.00705	0.00978	0.06154*	0.06399*	0.01516	0.04074*
ω_0 (Variance)	0.06082	0.02873	0.00108	0.01324	0.08575*	0.01615	0.12007*	0.01886**
d (Long memory)	0.42626*	0.16768*	0.89387*	0.29826*	0.55839*	0.42627*	0.32788*	0.99026*
φ_1 (ARCH)	0.38687*	0.15034	0.04031	0.40898*	0.07267	0.15079*	0.32496*	−0.02678
β_1 (GARCH)	0.70741*	0.29519*	0.94129*	0.66969*	0.58659*	0.57743*	0.58444*	0.92644*
Log ($\tilde{\alpha}$) (Hyperbolic)	0.02349	0.319786	−0.00096	0.06222	−0.02274	0.03049	0.00801	−0.00945
v (Tail)	7.18393*	7.818318*	9.432968*	5.570554*	9.09539*	6.70200*	5.80608*	4.27485*
LL	−9145.692	−9751.14	−3751.026	−3804.27	−7068.72	−6540.728	−7776.884	−5982.67
AIC	4.393995	4.684659	1.802843	1.828385	3.463398	3.196353	3.753078	2.857667
Q^2 (20)	19.84	31.6*	8.56	18.31	48.7*	40.8*	32.5*	22.4
ARCH (20)	0.9961	1.6348*	0.43198	0.9109	2.4578*	1.8743*	1.6048*	1.0800
RBD (20)	16.9125	4.21621	8.60776	−71.465	12.0259	20.4087	−42.3095	−1.49189
HYGARCH model with skewed student t distribution assumption								
μ (Mean)	0.04614	0.04169	0.00423	0.00429	0.04448*	0.04422*	0.00581	0.03725*
ω_0 (Variance)	0.05673	−0.00948	0.00105	0.01428	0.08744*	0.01521	0.11773*	0.01886**
d (Long memory)	0.42482*	0.15369*	0.89144*	0.30401*	0.57744*	0.41963*	0.33086*	0.99072*
φ_1 (ARCH)	0.38345*	0.13733	0.04164	0.40724*	0.07646	0.14946*	0.32681*	−0.02741
β_1 (GARCH)	0.70464*	0.27328**	0.94112*	0.66993*	0.60070*	0.56763*	0.58859*	0.92656*
Log ($\tilde{\alpha}$) (Hyperbolic)	0.02453	0.37008	−0.00086	0.05178	−0.02981	0.02681	0.00774	−0.00948
v (Tail)	7.306440*	7.88713*	9.46883*	5.61025*	10.0944*	7.26867*	5.81809*	4.28036*
ζ (Asymmetry)	−0.056693*	−0.06249*	−0.02794	−0.03398	−0.08781*	−0.08763*	−0.02443	−0.00948
LL	−9142.389	−9747.136	−3750.142	−3802.987	−7060.87	−6532.29	−7776.197	−5982.548
AIC	4.39289	4.683214	1.802898	1.828250	3.460043	3.19272	3.753229	2.858086
Q^2 (20)	19.98	31.7*	8.48	18.28	44.8*	40.9*	32.4*	22.5
ARCH (20)	1.0013	1.6383*	0.42825	0.90877	2.2582*	1.8733*	1.5967*	1.0824
RBD (20)	19.2023	3.28573	8.45248	95.1525*	12.7240	16.506	75.5377*	−1.60108

*and ** denote the 5% and 10% significance levels, respectively. LL is the value of maximised log likelihood. AIC is the Akaike (1974) information criterion.
For normal distribution assumption, robust standard errors are estimated with sandwich formula

compared with the observed returns, which represent the realised VaR. Both results are recorded for later assessment using the Kupeic (1995) likelihood ratio unconditional coverage (*LRuc*) test, which is defined as follows:

$$LR_{uc} = 2*\ln\left[(1-f)^{T-N} f^{N}\right] - 2*\ln\left[(1-\alpha)^{T-N}\alpha^{N}\right] \sim \chi^2_{(1)} \quad (19)$$

Where T is the sample size, N is the number of exceptions, f is the exception rate (N/T), and (1-α) is the confidence level. Further, H_0: $f = \alpha$ is the null hypothesis, which is tested against the alternative H_0: $f \neq \alpha$ hypothesis, where refers to the expected exception rate.

3. RESULTS

Descriptive statistics for the return series are shown in Table 2. Panel A shows that all return series have a positive average except for JPY/USD and NIKKEI 225 stock market index, for which they are negative. Standard deviations reveal that NYHCGR has the highest volatility, followed by WTI, at 2.8142 and 2.5095, respectively. However, exchange rate return volatilities are found to be lowest. Additionally, all series exhibit a statistically significant negative skewness, except for EUR/USD, meaning that the left tails are longer than the right tails in nearly all series. Similarly, in all cases it is shown that return series exhibit statistically significantly higher kurtosis especially for copper, NYHCGR, NIKKEI 225 stock market index, and gold, suggesting that these financial variables' return series have fat-tailed distributions. Thus, the Jarque-Bera (JB) test rejects the null hypothesis of normal distribution for each return series. Engel's (1982) ARCH test and the Ljung-Box Q test applied to squared return series using 12 lags indicate significant ARCH effect (Panel B). Results from the Augmented Dickey Fuller (ADF) and Philips Perron (PP) unit root tests, along with the Kwiatkowski, Philips, Schmidt, and Shin (KPSS) stationary test, show that all return series are stationary (Panel C). Lo's (1991) modified R/S

Table 5: FIAPARCH model estimation results

Model	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
FIAPARCH model with standard normal distribution assumption								
μ (Mean equation)	0.02333	0.03845	0.00389	0.00452	0.00396	0.01916	−0.00339	0.04628*
ω_0 (Variance equation)	0.1489	0.37852*	0.00320	0.00764*	0.13610*	0.07254*	0.17498**	0.07664*
d (Long memory)	0.41617*	0.36393*	0.90749*	0.98837*	0.53869*	0.30324*	0.27015*	0.40696*
φ_1 (ARCH)	0.41621*	0.18815*	0.03018	0.03479	0.14816*	0.21286*	0.12901	0.28828*
β_1 (GARCH)	0.68866*	0.44634*	0.94712*	0.94641*	0.59930*	0.46686*	0.31065	0.60410*
γ (APARCH asymmetry)	0.26061*	0.01474	0.09378	0.21156**	0.47805*	0.66009*	0.05370	−0.06004
δ (APARCH power)	1.76389*	1.95224*	1.59673*	1.18291*	1.12911*	1.53540*	2.14963*	1.85316*
LL	−9231.28	−9815.57	−3785.67	−3933.37	−7076.32	−6561.028	−7897.68	−6211.723
AIC	4.435084	4.715586	1.819462	1.890318	3.467119	3.206262	3.811322	2.966948
Q^2 (20)	15.60	23.67	8.72	12.61	34.36*	29.2*	20.2	9.29
ARCH (20)	0.79673	1.2207	0.44945	0.65725	1.6924*	1.5251**	1.0234	0.46993
RBD (20)	3.77291	9.83139	8.96586	9.48236	15.2966	89.2465*	−33.4305	7.81147
FIAPARCH model with student t distribution assumption								
μ (Mean equation)	0.05096**	0.06154**	0.00640	0.00908	0.02645	0.04271*	0.00894	0.04239*
ω_0 (Variance equation)	0.10558*	0.35562*	0.00242	0.00626**	0.17214*	0.04130	0.13188*	0.01838*
d (Long memory)	0.43212*	0.32801*	0.90677*	0.99683*	0.41473*	0.26984*	0.41123*	0.98723*
φ_1 (ARCH)	0.38322*	0.21301*	0.02536	0.00320	0.12842*	0.20334*	0.30677*	−0.04661
β_1 (GARCH)	0.70571*	0.46030*	0.94322*	0.95085*	0.47681*	0.44300*	0.64912*	0.94080*
γ (APARCH asymmetry)	0.30698*	0.102123	0.05224	0.08893	0.63636*	0.74328**	0.17277*	−0.3755**
δ (APARCH power)	1.73934*	1.943041*	1.73455*	1.30753*	1.15561*	1.56984*	1.68289*	0.93838
v (Tail)	7.520536*	8.06455*	9.847497*	5.68938*	10.09554*	7.99976*	5.92356*	4.33618*
LL	−9135.866	−9753.121	−3749.42	−3796.039	−7029.39	−6506.374	−7771.91	−5965.132
AIC	4.389758	4.686088	1.802553	1.824917	3.444637	3.180070	3.751162	2.849777
Q^2 (20)	19.38	43.30*	9.09	12.81	43.14*	28.7**	38.2*	92.6*
ARCH (20)	0.9880	2.1964*	0.46113	0.65471	2.0896*	1.4512**	1.8599*	4.4612*
RBD (20)	−16.974	5.63923	6.96243	5.57060	−18.1191	90.728*	8.1598	5.26566
FIAPARCH model with skewed student t distribution assumption								
μ (Mean equation)	0.02739	0.03826	0.00376	0.00429	0.00852	0.02378	−0.00112	0.03739*
ω_0 (Variance equation)	0.10961*	0.35194*	0.00243	0.00647**	0.17869*	0.04754**	0.13150*	0.01816*
d (Long memory)	0.43163	0.31980*	0.90508*	0.99322*	0.42160*	0.28260*	0.41412*	0.98850*
φ_1 (ARCH)	0.38022*	0.21042*	0.02539	0.00695	0.12908*	0.19006*	0.30757*	−0.04748
β_1 (GARCH)	0.70332*	0.45016*	0.94308*	0.94956*	0.48236*	0.44715*	0.65196*	0.94123*
γ (APARCH asymmetry)	0.33005*	0.11568	0.04985	0.08833	0.63071*	0.68065*	0.17408*	−0.3711**
δ (APARCH power)	1.722632*	1.96442*	1.73466*	1.30469*	1.12900*	1.59948*	1.68347*	0.94017
v (Tail)	7.722091*	8.12521*	9.89318*	5.71178*	10.99446*	8.68245*	5.93944*	4.33952*
ζ (Asymmetry)	−0.06595*	−0.06019*	−0.02724	−0.03029	−0.09526*	−0.09350*	−0.02662	−0.01283
LL	−9131.366	−9749.287	−3748.560	−3795.02	−7020.13	−6496.74	−7771.088	−5964.915
AIC	4.388078	4.684727	1.802622	1.824908	3.440593	3.175856	3.751248	2.850150
Q^2 (20)	19.40	44.18*	9.07	12.79	43.59*	28.7**	37.7*	92.2*
ARCH (20)	0.98810	2.2425*	0.46017	0.65296	2.1113*	1.4339**	1.8343*	4.4457*
RBD (20)	−28.7996	5.93796	6.55677	5.16328	−26.8628	48.1139*	8.2365	5.14928

*and ** denote the 5% and 10% significance levels, respectively. LL is the value of maximised log likelihood. AIC is the Akaike (1974) information criterion. For normal distribution assumption, robust standard errors are estimated with sandwich formula

test statistics analyses the long memory properties of the return and volatility series, indicating that all the series exhibit long memory properties in their volatility but short memory properties in their return (Panel D).

The estimation results of the FIGARCH model under standard normal, student, and skewed student t distribution assumptions, shown in Table 3, indicate that ARCH and GARCH parameters are positive and statistically significant in all cases except for EUR/USD, TSEC weighted stock index, and copper, where ARCH parameters are found to be insignificant. Fractional difference parameters d, taking values ranging from 0.3143 to 0.8892, are significant at the 5% significance level in all cases and generally highest for EUR/USD (nearly 0.88) and lowest for NYHCGR, copper, and JPY/USD, meaning that a shock to volatility will last longest for EUR/USD to decay and lowest for NYHCGR, copper, and JPY/USD.

The HYGARCH model results, presented in Table 4, reveal that GARCH parameters are positive and statistically significant in all cases at conventional significance levels, with the exception of copper when the HYGARCH model under normal distribution assumption is employed. In contrast, ARCH parameters are found to be statistically insignificant in most cases. Fractional difference parameters d are found to be positive and significant in all cases, and range from 0.1536 to 0.9907, and are generally highest for EUR/USD and gold, and lowest for NYHCGR. Besides, it is found that hyperbolic parameters Log(a) are not statistically significant at conventional significance levels in all cases, implying that GARCH components are covariance stationary.

Table 6: Long-memory GARCH-type Var model performance based on LRuc statistics for downside market risk

Quantile	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
FIGARCH model with standard normal distribution assumption								
0.0500	0.77024	0.56645	0.54176	0.31529	0.10025	0.98830	0.48281	0.58652
0.0250	0.24114	0.24114	0.31319	0.31319	0.00409*	0.47714	0.41422	0.24696
0.0100	0.13898	0.00231*	0.13446	0.01040*	0.00196*	0.00399*	0.22405	0.02311*
0.0050	0.04863*	8.96E−05*	0.21174	0.10442	0.00026*	4.91E−06*	0.39129	0.00294*
0.0025	0.06071**	0.00148*	0.16120	0.01923*	4.04E−07*	3.6E−06*	0.16120	0.00152*
FIGARCH model with student t distribution assumption								
0.0500	0.47493	0.39263	0.45237	0.97101	0.02894*	0.24632	0.54176	0.68771
0.0250	0.32594	0.32594	0.82059	0.53378	0.00701*	0.47714	0.53378	0.56480
0.0100	0.36211	0.02226*	0.75834	0.75834	0.00956*	0.01805*	0.75834	0.76427
0.0050	0.66386	0.10712	0.64999	0.65547	0.10019	0.00076*	0.64999	0.63604
0.0025	0.38261	0.38261	0.37793	0.37793	0.37047	0.01758*	0.28257	0.76390
FIGARCH model skewed student t distribution assumption								
0.0500	1.00000	0.56645	0.63979	0.68652	0.10025	0.65243	0.97101	0.68771
0.0250	0.84050	0.42927	0.82059	0.53378	0.04743*	0.47714	0.67032	0.85646
0.0100	0.31356	0.02226*	0.52007	0.52007	0.03856*	0.01805*	0.32083	0.76427
0.0050	1.00000	0.21625	0.64999	0.99104	0.38079	0.00661*	0.64999	0.32841
0.0025	0.38261	0.38261	0.75271	0.37793	0.37047	0.05522**	0.28257	0.76390
FIAPARCH model with standard normal distribution assumption								
0.0500	0.77024	0.47493	0.54176	0.79796	0.17097	0.30003	0.48281	0.49336
0.0250	0.68916	0.12215	0.16560	0.23072	0.00409*	0.92635	0.41422	0.03836*
0.0100	0.04311*	0.00041*	0.13446	0.02124*	6.33E−06*	0.01805*	0.22405	0.00538*
0.0050	0.39791	0.00030*	0.21174	0.00757*	3.28E−07*	0.00076*	0.21174	0.00099*
0.0025	0.06071**	7.55E−05*	0.05943**	0.00034*	0.001352*	1.19E−05*	0.16120	0.00153*
FIAPARCH model with student t distribution assumption								
0.0500	0.77305	0.25710	0.37240	0.79796	0.10025	0.87223	0.79796	0.21446
0.0250	0.83844	0.08366**	0.67032	0.31319	0.00701*	0.91063	0.67032	0.05868**
0.0100	0.53773	0.01096*	0.75834	0.98730	0.00013*	0.06712**	0.32083	0.23585
0.0050	1.00000	0.10712	0.65547	0.99104	0.04494*	0.00661*	0.33772	0.63604
0.0025	0.38261	0.06071**	0.37793	0.37793	0.37047	0.01758*	0.28257	0.76390
FIAPARCH model skewed student t distribution assumption								
0.0500	0.46085	0.56645	0.63979	0.48281	0.21843	0.37772	0.48281	0.21446
0.0250	0.68141	0.17385	0.67032	0.53378	0.03051*	0.92635	0.85817	0.08756**
0.0100	0.31356	0.13898	0.52007	0.98730	0.00196*	0.20274	0.32083	0.36916
0.0050	0.64222	0.21625	0.64999	0.99104	0.64208	0.00661*	0.33772	0.63604
0.0025	0.75893	0.06071**	0.75271	0.37793	0.75793	0.01758*	0.00000*	0.76390
HYGARCH model with standard normal distribution assumption								
0.0500	1.00000	0.15895	0.54176	0.31529	0.07510**	0.75956	0.48281	0.58652
0.0250	0.24114	0.03638*	0.23072	0.23072	0.00071*	0.36508	0.31319	0.08756**
0.0100	0.07943**	6.19E−05*	0.13446	0.01040*	5.01E−05*	0.00399*	0.13446	0.00538*
0.0050	0.00788*	2.51E−05*	0.39129	0.10442	0.00026*	4.9E−06*	0.10442	0.00099*
0.0025	0.01974*	7.54E−05*	0.16120	0.01923*	1.32E−05*	5.7E−08*	0.16120	0.00152*
HYGARCH model with student t distribution assumption								
0.0500	0.39263	0.02657*	0.45237	0.85628	0.02046*	0.15063	0.54176	0.49336
0.0250	0.32594	0.02312*	0.67032	0.53378	0.00409*	0.36508	0.53378	0.44150
0.0100	0.53773	0.00041*	0.98730	0.75834	0.00444*	0.00869*	0.75834	0.54642
0.0050	0.66386	0.00788*	0.99104	0.65547	0.04494*	0.00076*	0.64999	0.63604
0.0025	0.38261	0.06071**	0.37793	0.37793	0.37047	0.00497*	0.28257	0.76390
HYGARCH model skewed student t distribution assumption								
0.0500	0.88427	0.06933**	0.85628	0.58082	0.10025	0.55261	0.97101	0.58652
0.0250	0.84050	0.03638*	0.97977	0.53378	0.04743*	0.47714	0.67032	0.70432
0.0100	0.51016	0.00231*	0.52007	0.52007	0.03859*	0.01805*	0.32083	0.76427
0.0050	1.00000	0.02034*	0.64999	0.99104	0.64208	0.00076*	0.64999	0.63604
0.0025	0.38261	0.16391	0.75271	0.37793	0.37047	0.01758*	0.28257	0.76390

* and ** denote the 5% and 10% significance level, respectively. The figures are the probability values of the Kupeic (1995) test

As with the previous two models, the FIAPARCH model estimation results (Table 5) also indicate that fractional difference parameters d, ranging from 0.2698 to 0.9968, are positive and significant at the 5% significance level for each return series. Besides, the power term, δ with a value that ranges from 0.9384 to 2.1496, is found to be significant at the 10% or better significance level in all cases, with the exception of gold, for which it is found to be statistically insignificant under student and skewed student t distribution assumptions. For asymmetry parameters, γ is found to be positive in all cases except for gold, for which it is negative.

Table 7: Long-memory GARCH-type VaR model performance based on LRuc statistics for upside market risk

Quantile	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
FIGARCH model with standard normal distribution assumption								
0.9500	0.23316	0.00311*	0.14259	0.39396	0.42639	0.00046*	0.48281	0.28652
0.9750	0.13588	0.20474	0.69983	0.41422	0.57860	0.00473*	0.53378	0.56480
0.9900	0.31356	0.53773	0.35339	0.22405	0.53619	0.03506*	0.74218	0.01141*
0.9950	1.00000	0.66386	0.21174	0.00272*	0.64208	0.13691	0.65547	0.67059
0.9975	0.75893	0.06071**	0.16120	0.00034*	0.74274	0.76853	0.74860	0.16611
FIGARCH model with student t distribution assumption								
0.9500	0.55655	0.00529*	0.10468	0.91292	0.95919	0.00164*	0.97101	0.97688
0.9750	0.01524*	0.08572**	0.55164	0.67032	0.15310	0.00204*	0.21352	0.70432
0.9900	0.07859**	0.51016	0.52007	0.35339	0.33275	0.01068*	0.03113*	0.16601
0.9950	0.02851*	0.66386	0.99104	0.65547	0.34619	0.13691	0.02918*	0.32841
0.9975	0.27947	0.75893	0.28257	0.75271	0.75793	0.76853	0.28257	0.00000*
FIGARCH model with skewed student t distribution assumption								
0.9500	1.00000	0.04842*	0.39396	0.85628	0.41740	0.09659**	0.97101	0.79548
0.9750	0.13588	0.20474	0.69983	0.41422	0.89005	0.01012*	0.55164	0.70432
0.9900	0.16963	0.53773	0.98730	0.35339	0.77754	0.03506*	0.03113*	0.16601
0.9950	0.12578	0.66386	0.39129	0.39129	0.64208	0.13691	0.12823	0.63604
0.9975	0.27947	0.16391	0.75271	0.37793	0.75793	0.76853	0.28257	0.00000*
FIAPARCH model with standard normal distribution assumption								
0.9500	0.23316	0.00311*	0.24726	0.74519	0.92453	4.29E−06*	0.58082	0.35952
0.9750	0.20474	0.20474	0.85817	0.41422	0.72982	9.14E−05*	0.31319	0.44150
0.9900	0.51016	0.53773	0.35339	0.00486*	0.77754	0.08999**	0.74218	0.01141*
0.9950	0.64222	0.39791	0.21174	0.00091*	0.64208	0.13691	0.99104	0.40324
0.9975	0.74281	0.06071**	0.01923*	0.00034*	0.05742**	0.76853	0.74860	0.16611
FIAPARCH model with student t distribution assumption								
0.9500	0.23316	0.03286**	0.24726	0.63979	0.59817	0.00089*	0.48281	0.58652
0.9750	0.01524*	0.20474	0.69983	0.53378	0.44213	9.14E−05*	0.09015**	0.33634
0.9900	0.07859**	0.74647	0.52007	0.35339	0.33275	0.00232*	0.03113*	0.50233
0.9950	0.02851*	0.66386	0.65547	0.39129	0.64208	0.13691	0.02918*	0.99286
0.9975	0.27947	0.16391	0.74860	0.37793	0.15690	0.29338	0.28257	0.27701
FIAPARCH model with skewed student t distribution assumption								
0.9500	0.66029	0.17827	0.48281	0.54176	0.13188	0.00830*	0.68652	0.49336
0.9750	0.40496	0.40496	0.85817	0.31319	0.89005	0.00080*	0.41899	0.33634
0.9900	0.31356	0.75444	0.52007	0.35339	0.77754	0.03506*	0.03113*	0.50233
0.9950	0.02851*	0.66386	0.39129	0.21174	0.64208	0.13691	0.02918*	0.99286
0.9975	0.27947	0.16391	0.75271	0.37793	0.05742**	0.76853	0.28257	0.27701
HYGARCH model with standard normal distribution assumption								
0.9500	0.29854	0.04842*	0.07504**	0.48281	0.62084	0.00046*	0.58082	0.35952
0.9750	0.20474	0.20474	0.85817	0.41422	0.57860	0.01012*	0.31319	0.56480
0.9900	0.31356	0.53773	0.74218	0.22405	0.53619	0.03506*	0.74218	0.01141*
0.9950	1.00000	0.39791	0.21174	0.00272*	0.64208	0.13691	0.39129	0.67059
0.9975	0.74281	0.06071**	0.05943**	0.00143*	0.74274	0.76853	0.74860	0.06174**
HYGARCH model with student t distribution assumption								
0.9500	0.46085	0.29854	0.10468	0.91292	0.81054	0.00498*	0.97101	0.68771
0.9750	0.01524*	0.20474	0.55164	0.67032	0.22820	0.00473*	0.21352	0.56480
0.9900	0.07859**	0.75444	0.52007	0.35339	0.33275	0.01068*	0.03113*	0.16601
0.9950	0.02851*	0.66386	0.99104	0.39129	0.34619	0.13691	0.02918*	0.63604
0.9975	0.27947	0.16391	0.28257	0.75271	0.75793	0.76853	0.28257	0.00000*
HYGARCH model with skewed student t distribution assumption								
0.9500	0.88427	0.55655	0.31529	0.74519	0.50324	0.30003	0.97101	0.68771
0.9750	0.13588	0.29430	0.69983	0.41422	0.94728	0.01012*	0.97977	0.44150
0.9900	0.16963	0.36211	0.75834	0.35339	0.77754	0.03506*	0.08106**	0.16601
0.9950	0.12578	0.66386	0.65547	0.39129	0.64208	0.13691	0.12823	0.63604
0.9975	0.27947	0.06071**	0.75271	0.37793	0.75793	0.76853	0.28257	0.00000*

* and ** denote the 5% and 10% significance level, respectively. The figures are the probability values of the Kupeic (1995) test

However, it is generally statistically significant at conventional significance levels for WTI, NIKKEI 225 stock market index, TSEC weighted stock index, copper, and gold only, meaning that negative shocks have more impact on conditional volatility than positive shocks of equal magnitude for those financial variables. The only exception is gold, for which the opposite linkage is identified.

As for student t and skewed student t distribution assumptions, estimated tail parameters are statistically significant in all cases at

Table 8: FHS and HS models' VaR performances based on LRuc statistics for upside and downside market risk

Quantile	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
Long trading position (Downside market risk)								
FHS								
0.0500	0.2985	0.8850	0.7980	0.4828	0.4174	0.0475*	0.9129	0.7955
0.0250	1.0000	0.8384	0.6998	0.5338	0.1530	0.6098	0.5516	0.8565
0.0100	0.7465	0.2306	0.9873	0.7583	0.0096*	0.2027	0.5201	0.9899
0.0050	0.6422	1.0000	0.3913	0.6500	0.0186*	0.6769	0.1282	0.6360
0.0025	0.7589	0.3826	0.3779	0.7486	0.7579	0.7685	0.0000*	0.7639
HS								
0.0500	0.5665	0.0217*	0.0750**	0.6865	0.7011	0.0000*	0.0000*	0.0654**
0.0250	0.4293	0.0512**	0.0902**	0.8582	0.5068	0.0000*	0.0000*	0.0490*
0.0100	0.1696	0.3136	0.0811**	0.5269	0.3328	0.0003*	0.0000*	0.0019*
0.0050	0.1258	0.3325	0.0292*	0.6555	0.3462	0.0316*	0.0000*	0.1238
0.0025	0.2795	0.7428	0.2826	0.7486	0.7579	0.0000*	0.0000*	0.7382
Short trading position (Upside market risk)								
FHS								
0.9500	0.6603	0.0031*	0.2473	0.1481	0.1710	0.0134*	0.6865	0.8615
0.9750	0.1359	0.0857**	0.8582	0.2307	0.6405	0.0020*	0.8582	0.3363
0.9900	0.3136	0.7465	0.2240	0.2240	0.7226	0.0351*	0.5201	0.0018*
0.9950	0.6422	0.3979	0.3913	0.0196*	0.3808	0.3559	0.3377	0.6706
0.9975	0.2795	0.7428	0.7486	0.1612	0.1569	0.7685	0.2826	0.1662
HS								
0.9500	0.6663	0.0140*	0.1047	0.5408	0.5032	0.0000*	0.0000*	0.0202*
0.9750	0.2411	0.2943	0.9798	0.8582	0.7298	0.0000*	0.0000*	0.0275*
0.9900	0.7544	1.0000	0.3208	0.7422	0.5362	0.0023*	0.0003*	0.3078
0.9950	1.0000	0.6639	0.3377	0.9910	0.9766	0.0000*	0.0292*	0.6360
0.9975	0.7428	0.7428	0.2826	0.7527	0.7427	0.0000*	0.0000*	0.2770

* and ** denote the 5% and 10% significance level, respectively. The figures are the probability values of the Kupeic (1995) test

the 5% significance level, indicating that the fat-tail phenomenon is valid for all the relevant financial variable return series. Additionally, asymmetric parameters are found to be negative in all cases. In particular, it is statistically significant for only WTI, NYHCGR, NIKKEI 225 stock market index, and TSEC weighted stock index. Concerning diagnostic checks, Engel's (1982) ARCH test, Tse's (2002) residual-based diagnostics (RBD) test, and the Ljung-Box Q test, all of which have the null hypothesis of "no ARCH effect", are used to analyse whether or not the ARCH effect is eliminated. Results show that in most cases, the models are not capable of capturing the ARCH effect, especially for the NIKKEI 225 stock market index, TSEC weighted stock index, copper, and NYHCGR under student t and skewed student t distribution assumptions.

3.1. Evaluating out-of-sample market risk forecasting performances of alternative models

In this subsection, under the normal, student t and skewed student t distribution assumptions, the FIGARCH, FIAPARCH, and HYGARCH models' out-of-sample one-day-ahead VaR performances are compared with the FHS and HS models' performances for each of the financial variables. VaR results are presented in Tables 6-8, whereas ES values are reported in Tables 9-11 for each model. Additionally, as an example, graphs of the out-of-sample VaR forecasts of the FHS and HS models together with observed returns (which represent the realised VaR) are presented in Figures 2 and 3, respectively.

First, it is concentrated on long-memory GARCH-type model performances only. Based on the LRuc test statistic, results indicate that at the 10% or better significance level for downside risk, the FIGARCH model with skewed student t distribution assumption is the most appropriate model since it has fewer rejections (6) out of a total of 40 cases. This is followed by the FIGARCH model with student t distribution assumption and the FIAPARCH model with skewed student t distribution assumption, both of which have seven rejections. The worst performing models are the HYGARCH model with normal distribution assumption, because it has the highest rejections (21), followed by the FIAPARCH model with standard normal distribution assumption with 20 rejections. Additionally, among the alternative distribution assumptions, the standard normal distribution assumption is found to be the worst one in all cases, whereas the skewed student t distribution assumption is the most appropriate model, followed by the student t distribution assumption.

For upside market risk, however, the findings reveal that the most appropriate model is the HYGARCH with skewed student t distribution assumption, since it only has five rejections. This is followed by the FIGARCH model with skewed student t distribution assumption with six rejections. In contrast, the FIAPARCH and HYGARCH models with standard normal distribution assumptions are found to be the worst models due to the fact that each of them has 11 rejections, followed by the FIAPARCH model with student t distribution assumption with 10 rejections. Besides, the skewed student t distribution assumption is found to be the most appropriate distribution without any exception, although it is observed that both standard normal and student t distribution assumptions perform poorly.

Table 9: Long-memory GARCH-type model ES values for long trading position (%)

Quantile	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
FIGARCH model with standard normal distribution assumption								
0.0500	−4.6353	−5.4568	−1.1755	−1.5025	−3.0864	−1.8587	−2.7604	−2.3517
0.0250	−5.1870	−6.4403	−1.2931	−1.6662	−3.4760	−2.2263	−2.9912	−2.5922
0.0100	−5.2096	−7.4875	−1.3773	−1.9232	−3.8469	−2.4086	−3.3666	−3.0013
0.0050	−5.7584	−8.1627	−1.5696	−2.3698	−3.9054	−2.4950	−3.5538	−3.3002
0.0025	−6.4485	−10.161	−1.6187	−2.5138	−3.7876	−2.5930	−3.7154	−3.7275
FIGARCH model with student t distribution assumption								
0.0500	−4.4888	−5.3931	−1.1798	−1.4632	−3.0786	−1.7650	−2.6669	−2.3367
0.0250	−5.1981	−6.4900	−1.3302	−1.7109	−3.4750	−2.2263	−3.0079	−2.6479
0.0100	−5.2219	−8.0134	−1.5449	−2.3698	−4.0334	−2.4950	−3.5790	−3.4140
0.0050	−6.4485	−10.161	−1.6999	−2.6039	−4.2759	−2.5727	−3.8724	−5.3330
0.0025	−7.6078	−13.816	−1.6999	−2.7617	−5.8978	−2.7464	−4.8174	−6.2703
FIGARCH model with skewed student t distribution assumption								
0.0500	−4.5464	−5.4568	−1.1816	−1.4874	−3.1415	−1.7970	−2.7018	−2.3367
0.0250	−5.3108	−6.5870	−1.3298	−1.7109	−3.4557	−2.2263	−3.0329	−2.6837
0.0100	−5.9752	−8.0134	−1.5696	−2.4102	−3.8710	−2.4950	−3.6277	−3.4140
0.0050	−7.0443	−10.722	−1.6999	−2.6343	−4.6093	−2.6698	−3.8724	−6.2703
0.0025	−7.6078	−13.816	−1.6741	−2.7617	−5.8978	−2.7934	−4.8174	−6.2703
FIAPARCH model with standard normal distribution assumption								
0.0500	−4.7810	−5.4077	−1.1857	−1.4615	−3.1877	−1.9702	−2.7604	−2.3467
0.0250	−5.3705	−6.1480	−1.3095	−1.6633	−3.4582	−2.2934	−2.9912	−2.6064
0.0100	−5.5972	−7.0749	−1.4822	−1.9559	−3.6069	−2.4470	−3.3666	−2.9292
0.0050	−5.5972	−8.3795	−1.5696	−2.1029	−3.7880	−2.5221	−3.4687	−3.2240
0.0025	−6.4485	−9.0861	−1.6528	−2.2697	−4.2653	−2.6158	−3.7154	−3.7275
FIAPARCH model with student t distribution assumption								
0.0500	−4.7017	−5.2468	−1.1767	−1.4550	−3.1501	−1.8346	−2.7343	−2.2913
0.0250	−5.4567	−6.0975	−1.3246	−1.6651	−3.5001	−2.2496	−2.9991	−2.6554
0.0100	−5.9729	−7.6326	−1.5449	−2.2560	−3.6856	−2.5265	−3.6277	−3.3401
0.0050	−7.0443	−10.161	−1.6528	−2.6040	−4.1389	−2.6158	−4.0177	−5.3330
0.0025	−7.6078	−11.477	−1.6999	−2.7617	−5.8718	−2.7782	−4.8174	−6.2703
FIAPARCH model with skewed student t distribution assumption								
0.0500	−4.8727	−5.4258	−1.1917	−1.4853	−3.2016	−1.9543	−2.7861	−2.2913
0.0250	−5.5416	−6.2252	−1.3246	−1.7030	−3.5101	−2.2934	−3.0865	−2.6879
0.0100	−6.8203	−8.4828	−1.5696	−2.2560	−3.7828	−2.5221	−3.6277	−3.4271
0.0050	−7.6078	−10.433	−1.6999	−2.6040	−4.8727	−2.6158	−4.0177	−5.3330
0.0025	−8.4171	−11.477	−1.6741	−2.7617	−7.9251	−2.7782	na	−6.2703
HYGARCH model with standard normal distribution assumption								
0.0500	−4.5366	−5.1864	−1.1861	−1.5025	−3.0876	−1.8252	−2.7604	−2.3517
0.0250	−5.1870	−5.9329	−1.3248	−1.6485	−3.4404	−2.1970	−2.9695	−2.5985
0.0100	−5.0138	−6.7645	−1.4822	−1.9232	−3.7094	−2.4086	−3.2804	−2.9398
0.0050	−5.3155	−7.6482	−1.6206	−2.3698	−3.9054	−2.4950	−3.4888	−3.2685
0.0025	−5.9752	−9.0861	−1.7247	−2.5138	−4.0151	−2.5564	−3.7154	−3.7275
HYGARCH model with student t distribution assumption								
0.0500	−4.4426	−4.9699	−1.1798	−1.4504	−3.0731	−1.7247	−2.6669	−2.3381
0.0250	−5.1981	−5.8613	−1.3216	−1.7109	−3.4902	−2.1911	−3.0079	−2.6295
0.0100	−5.3155	−6.9355	−1.5007	−2.3698	−4.0005	−2.4373	−3.5790	−3.3135
0.0050	−6.4485	−8.6122	−1.7247	−2.6039	−4.2095	−2.5727	−3.8724	−5.3330
0.0025	−7.6078	−11.477	−1.6999	−2.7617	−5.8978	−2.7790	−4.8174	−6.2703
HYGARCH model with skewed student t distribution assumption								
0.0500	−4.5427	−5.0453	−1.1945	−1.4945	−3.1160	−1.7629	−2.7018	−2.3520
0.0250	−5.3108	−5.9092	−1.3358	−1.7109	−3.4557	−2.2263	−3.0329	−2.7158
0.0100	−5.7290	−7.1047	−1.5696	−2.4102	−3.8710	−2.4950	−3.6277	−3.4140
0.0050	−7.0443	−9.0861	−1.6999	−2.6343	−4.8906	−2.5727	−3.8724	−5.3330
0.0025	−7.6078	−11.868	−1.6741	−2.7617	−5.8978	−2.7464	−4.8174	−6.2703

"na" denotes that there is no exception, Which also indicates that the relevant model measures the real VaR more than it should

Turning to the FHS and HS models' one-day ahead out-of-sample forecasting VaR performances, the results show that the FHS model produces only four rejections whereas the HS model produces 19 rejections for downside market risk at the 10% or better significance level. For upside market risk, the findings indicate that the FHS model has seven rejections while the HS model has 13 rejections.

In this framework and taking all these findings together, the results show that among the alternative models examined in this study, the most appropriate model for downside market risk is the FHS model, to which the relevant literature has paid more attention. However, for upside market risk the HYGARCH model with skewed student t distribution assumption is the most appropriate.

Table 10: Long-memory GARCH-type model ES values for short trading position (%)

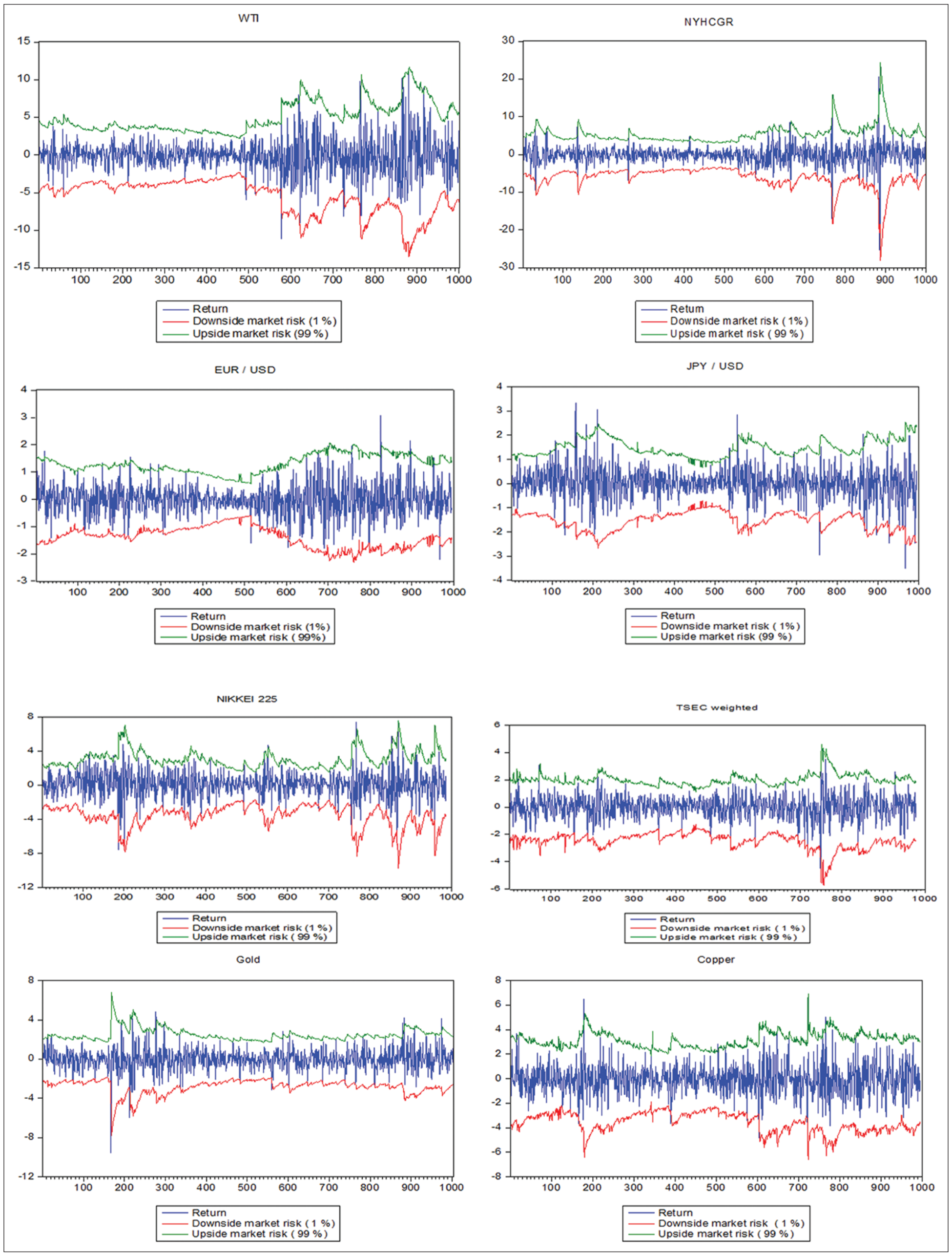
Quantile	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
FIGARCH model with standard normal distribution assumption								
0.9500	4.8347	6.2991	1.3123	1.4549	2.9728	1.8310	2.7814	2.4095
0.9750	6.1740	6.9283	1.4532	1.3814	3.4450	2.2968	3.0978	2.6337
0.9900	8.8210	7.9621	1.6140	1.4242	4.7778	2.7616	3.6889	2.9208
0.9950	8.6088	9.2207	1.8263	1.3159	4.8245	2.8202	4.2101	3.7036
0.9975	8.9075	9.2207	1.8694	1.2882	5.2092	2.8202	4.6716	3.9293
FIGARCH model with student t distribution assumption								
0.9500	4.8964	6.2367	1.3305	1.4117	2.9747	1.7715	2.7366	2.2460
0.9750	7.1119	7.3047	1.4817	1.7042	3.6048	2.3060	3.4542	2.7556
0.9900	8.6088	8.4185	1.8263	2.0180	5.1226	3.0527	4.5094	3.7036
0.9950	9.8140	9.2207	1.9606	2.6034	5.2092	2.8202	6.5152	4.2134
0.9975	9.8140	12.550	3.0643	2.8770	6.0693	2.8202	6.5152	na
FIGARCH model skewed student t distribution assumption								
0.9500	4.8694	5.8800	1.2730	1.3923	2.8726	1.7264	2.7366	2.2333
0.9750	6.1740	6.9283	1.4774	1.6728	3.3517	2.2126	3.3491	2.7556
0.9900	9.0555	7.9621	1.6692	2.0180	4.5086	2.7648	4.5094	3.7036
0.9950	8.9021	9.2207	1.8666	2.4414	4.8245	2.8202	4.6716	4.2092
0.9975	9.8140	9.9901	2.2519	2.6668	6.0693	2.8202	6.5152	na
FIAPARCH model with standard normal distribution assumption								
0.9500	4.9418	6.0082	1.2993	1.4437	2.8936	1.8339	2.7645	2.3862
0.9750	6.0916	6.9283	1.4791	1.4478	3.4572	2.3519	3.0377	2.6867
0.9900	8.7535	7.9621	1.6737	1.3529	4.1224	2.6252	3.6889	2.9208
0.9950	8.2068	8.7488	1.8263	1.3409	4.0791	2.8202	4.2518	3.6672
0.9975	8.9021	9.2207	1.8666	1.3492	4.0791	2.8202	4.6716	3.9293
FIAPARCH model with student t distribution assumption								
0.9500	5.1390	5.8311	1.3102	1.3768	2.8693	1.7953	2.7870	2.2518
0.9750	6.8249	6.9283	1.4774	1.6979	3.5337	2.3230	3.5275	2.6505
0.9900	8.2216	8.4012	1.8263	2.0281	3.9092	2.8202	4.5094	3.7300
0.9950	9.8140	9.2207	1.9594	2.4688	4.0791	2.8202	6.5152	4.2012
0.9975	9.8140	9.9901	2.6085	2.6668	4.3391	3.0525	6.5152	4.8387
FIAPARCH model with student t distribution assumption								
0.9500	4.9253	5.4768	1.2958	1.3661	3.7715	1.7327	2.7646	2.2428
0.9750	6.0577	6.7437	1.4791	1.6735	3.4256	2.1808	3.2727	2.6505
0.9900	8.5444	7.9583	1.8263	2.0281	4.1224	2.7616	4.5094	3.7300
0.9950	9.8140	9.2207	1.8666	2.3844	4.0791	2.8202	6.5152	4.2012
0.9975	9.8140	9.9901	2.2519	2.6668	4.0791	2.8202	6.5152	4.8387
HYGARCH model with standard normal distribution assumption								
0.9500	4.7666	5.6902	1.3402	1.4494	2.9776	1.8310	2.7738	2.3947
0.9750	5.9647	6.9283	1.2776	1.6859	3.4790	2.2126	3.0377	2.6337
0.9900	8.8210	7.9621	1.2588	1.9752	4.7778	2.7616	3.6889	2.9208
0.9950	8.6088	8.7488	1.2151	2.0180	4.8245	2.8202	3.9703	3.7036
0.9975	8.9021	9.2207	1.1613	2.2462	5.2092	2.8202	4.6716	3.7036
HYGARCH model with student t distribution assumption								
0.9500	4.9075	5.3443	1.3305	1.4117	2.9474	1.7386	2.7366	2.2319
0.9750	6.7319	6.9283	1.4817	1.7042	3.6371	2.2141	3.4542	2.7204
0.9900	8.6088	7.9583	1.8263	2.0180	5.1226	3.0527	4.5094	3.7036
0.9950	9.8140	9.2207	1.9606	2.4414	5.2092	2.8202	6.5152	4.2092
0.9975	9.8140	9.9901	3.0643	2.8770	6.0693	2.8202	6.5152	na
HYGARCH model with skewed student t distribution assumption								
0.9500	4.8537	5.2583	1.2846	1.3868	2.9001	1.6548	2.7366	2.2319
0.9750	6.1740	6.8210	1.4774	1.6728	3.3206	2.2126	3.1759	2.6983
0.9900	9.0555	7.5509	1.7720	2.0180	4.5086	2.7648	4.2518	3.7036
0.9950	8.9021	9.2207	1.9594	2.4414	4.8245	2.8202	4.6716	4.2092
0.9975	9.8140	9.2207	2.2519	2.6668	6.0693	2.8202	6.5152	na

"na" denotes that there is no exception, Which also indicates that the relevant model measures the real VaR more than it should

4. CONCLUSION

Since the 2007-2008 global financial crisis, traditional methods commonly used to measure market risk have become the subject of criticism, in large part because of their inability to meet accurately the market losses. As a result, new models have become the focus of close attention, with the goal of improving the VaR performances. In this regard, and in terms of market risk measurement, long-memory GARCH-type models have primarily emerged as a better choice than short-memory GARCH-type models.

Figure 2: Out-of-sample value-at-risk forecasts of the FHS model for downside and upside market risk

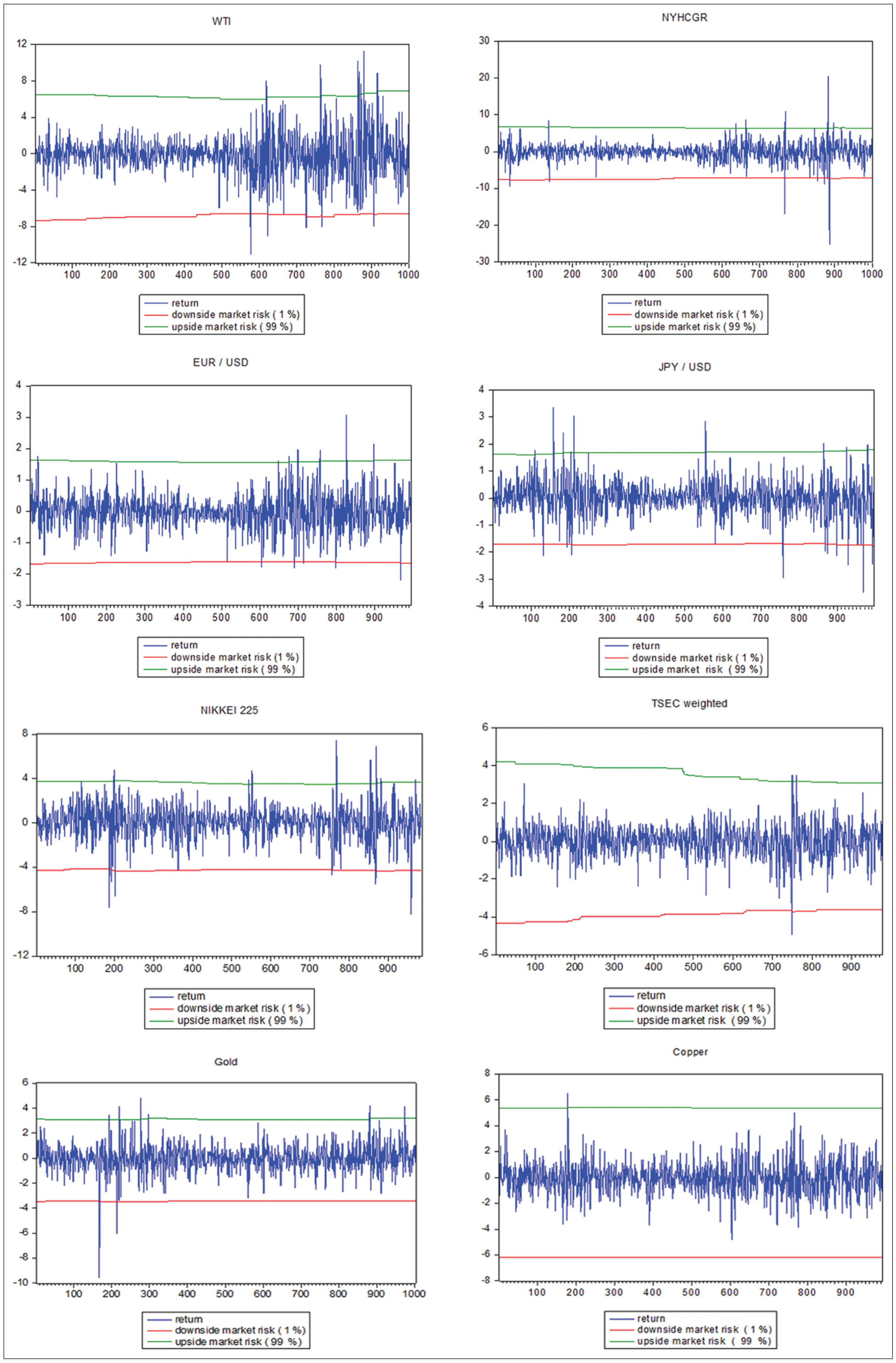


However, one of the main gaps in the relevant literature is that the one-day ahead out-of-sample VaR forecasting performance of the long-memory GARCH-type models has not been adequately compared with the performances of two other popular models commonly used by financial institutions: the FHS and HS models. In this regard, this study compares the performance of long-memory GARCH-type models with FHS and HS models in order to examine whether or not long-memory GARCH-type models also perform better than FHS and HS models for eight different financial variables (WTI, gasoline, EUR/USD, JPY/USD, NIKKEI 225 stock market index, TSEC weighted stock index, copper, and gold).

Our results clearly show that the FHS model should be used for long trading positions, whereas the HYGARCH model under skewed student t distribution assumption should be preferred for short trading positions. Additionally, findings also indicate that the worst models for downside market risk are the HYGARCH and FIAPARCH models under standard normal distribution assumptions, while it is the HS model that is worst for upside market risk. In this regard, the results presented by this study provide financial institutions and investors with important information about market risk measurement, variance forecasting, option pricing, asset allocations, and hedging decisions.

However, this study only compares the one-day ahead out-of-sample VaR forecasting performance of standard long-memory GARCH-type models (i.e. FIGARCH, HYGARCH, and FIAPARCH models) with standard FHS and HS models. However, some papers

Figure 3: Out-of-sample value-at-risk forecasts of the HS model for downside and upside market risk



in the extant literature report that newly-developed extensions of standard long-memory GARCH-type models (e.g. the adaptive FIGARCH model developed by Baillie and Morana (2009), and the time-varying FIGARCH model introduced by Belkhouja and Boutahary (2009)) have a better forecasting performance than the standard long-memory GARCH-type models. Therefore, adaptive- and time-varying FIGARCH model performances can also be compared with FHS and HS model performances. Additionally, instead of filtering FHS with a standard GARCH model with normal distribution, which is the common approach in the relevant literature, the HYGARCH or FIAPARCH models with skewed student t distribution assumption can also be used as filters for the FHS model, which in turn may lead to further improvements to the FHS model's forecasting performances. Moreover, since the backtesting procedure is one of the most important parts of VaR analysis, different backtesting procedures can also be employed to evaluate the models' performances. However, all these issues have been left for future studies.

Table 11: FHS and HS models' ES values (%)

Quantile	WTI	NYHCGR	EUR/USD	JPY/USD	NIKKEI225	TSEC	Copper	Gold
Long trading position (Downside market risk)								
FHS								
0.0500	−4.9142	−5.5734	−1.2024	−1.4900	−3.1975	−2.0440	−2.6683	−2.3834
0.0250	−5.2400	−7.1141	−1.3378	−1.7129	−3.6543	−2.3554	−3.0615	−2.7389
0.0100	−5.9341	−8.3669	−1.5007	−2.3053	−3.9643	−2.5727	−3.5675	−3.5744
0.0050	−7.6078	−12.958	−1.6206	−2.7617	−4.0974	−3.0480	−4.2590	−5.3330
0.0025	−8.4171	−13.816	−1.6999	−3.2240	−7.9251	−3.9213	na	−6.2703
HS								
0.0500	−5.2538	−6.6758	−1.3554	−1.5180	−3.5278	−2.7794	−3.4758	−2.7095
0.0250	−6.1276	−8.8408	−1.5566	−1.8708	−4.2131	−3.6786	−4.8175	−3.5015
0.0100	−8.5136	−12.068	−1.8534	−2.2525	−6.1327	−4.9569	na	−7.8099
0.0050	−10.088	−17.284	−2.1957	−2.6039	−7.4705	−4.9569	na	−7.8099
0.0025	−11.126	−21.163	−2.1957	−3.2240	−7.9251	na	na	−7.8099
Short trading position (Upside market risk)								
FHS								
0.9500	4.8930	6.2420	1.3264	1.2956	2.7634	1.7945	2.7467	2.3024
0.9750	6.5270	7.3580	1.4799	1.6385	3.2259	2.3654	3.0957	2.6758
0.9900	8.9448	8.4571	1.6670	1.9589	4.1947	2.4052	3.6824	3.0466
0.9950	9.2348	8.8576	1.8696	2.0602	4.4730	2.5733	4.3448	3.7036
0.9975	9.8140	14.513	2.6085	2.7456	5.0486	2.8202	6.5152	3.9293
HS								
0.9500	5.4883	6.3729	1.3937	1.4237	3.1209	2.7485	3.8764	2.6319
0.9750	6.5010	7.5086	1.5483	1.8346	3.8506	3.0169	5.1733	3.3024
0.9900	8.6563	9.3068	2.0375	2.2886	5.1781	3.5112	6.5152	3.9978
0.9950	9.8465	10.776	2.3953	2.7553	5.9172	na	6.5152	4.3361
0.9975	10.753	15.772	3.0643	3.0848	6.6826	na	na	4.8388

"na" denotes that there is no exception, Which also indicates that the relevant model measures the real VaR more than it should

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