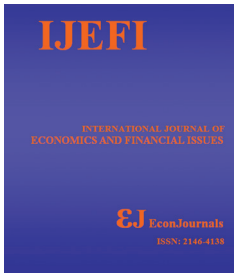

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Testing the Fit of Cobb-Douglass Production Function within Unrestricted Least Squares

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ABSTRACT

Criticisms trailed Cobb-Douglass production model and its constant returns to Scale assumption after its application in America Economy in 1927. Studies using restricted least squares approach to validate the model have produced different results in different economies. The question this paper tried to answer is this-without restriction, can Cobb-Douglass model still fit well in an economy's production? We answered this question using unrestricted least squares method with quarterly data generated from Central Bank of Nigeria between 2009 and 2012. The study is very important in Nigeria where the economy has not performed well despite heavy government investment in the last two decades. Result from the shows that even without restriction, Nigeria economy displays constant returns to scale production, suggesting that the Cobb-Douglass Production Function fits well in Nigeria economy. The result has an implication for a country in which the economy is driven by the public sector.

Keywords: Cobb-Douglass, Production Function, Constant Returns, Unrestricted Least Square
JEL Classification: E23

1. INTRODUCTION

The Cobb-Douglass theory of production has provided important framework for the measurement of productivity and employment of factors of production since 1930s. Cobb and Douglas have modelled the growth of output in American manufacturing sector between 1899 and 1922 in which output of goods were determined by combination of two factor inputs, namely labour and capital under the assumption of constant returns to scale production. Research attention given to the novel work of the Cobb-Douglass model in the last five decades is a worthwhile because of its significance to the macro and micro economy. Following the assumptions made in the model, the work of Cobb and Douglass attracted many criticisms and questions.

Fraser (2002) saw the omission of technical change in the specification of the Cobb-Douglass model as a serious limitation to the acceptance of its assumptions. By failure to recognize technical change, Fraser says Cobb and Douglass assumed that technology was constant within the period of their study, which does not hold in the real sense of it. The neoclassical economists also attacked the model on the bases that the productivity theory is more of an abstraction than a quantifiable. Douglass while responding to the criticisms noted that the critics were so hostile even to the extent of recommending for the work to be thrown into the waste basket and further research in it stopped. In the defence of the model, From criticism, Cobb-Douglass model started receiving research interest with positive comments and positive empirical result. Miller (2008) accepts that Cobb-Douglass model is very simple to use and can fit many data sets very well for empirical forecasting. Many studies have equally been done in developing as well as developed countries, trying to validate the Cobb-Douglass model. Results of such studies have differed, making it difficult to make a definite conclusion about the Cobb-Douglass postulation. Hence, up to this point, the applicability of constant returns to scale production as postulated by Cobb-Douglass is still at the centre of research interest around the globe.

Adetunji et al. (2012); Abidemi (2010) and some other studies have been done to validate the applicability of Cobb-Douglass laws of

production with Nigerian data. Adetunji et al. used macro data to study the application of laws of production as propounded by cobb-Douglass model. Their result shows that production function in Nigeria follows the constant returns to scale as predicted by cobb-Douglass model, which means that doubling input use will double output in the country. However, in a micro study by Abidemi (2010), result shows that in the banking sector, when inputs are doubled, output will more than double in the sector. It then suggests that in the banking industry in Nigeria increasing returns to scale prevails. This contradicting result between micro and macro studies is a source of worry.

The objective of this study is to remove the restriction imposed by the Cobb-Douglas model and find out if it can still fit the economy of Nigeria. The removal of the restriction is one of the steps taken to reconcile the macro and micro studies done in Nigeria so as to provide policy makers a more reliable insight of the application of the model in the country. In order to make the reconciliation, we decided to use study methodology which is different from the earlier ones. Previous studies used restricted least squares as empirical model. We applied Unrestricted least squares in a macro data and make suggestion that another study should apply the same unrestricted least squares in a micro data. Moreover, rather than use government investment as a proxy of total investment, we used gross fixed capital formation (GFCF) to represent the economy's total investment.

In order to achieve our objective, the remainder of the work is organised as follows: Section 11 is the theoretical framework and literature review; Section 111 describes the methodology used; Section 1V is the result and discussion of research findings and Section V is the conclusion the work.

2. THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1. Theoretical Framework

Economic scholars in the 19th century did not pay much attention to the mathematical modelling of the relationship between input and output. The physiocrats only recognised labour as a factor of production and treated capital as non-productive because they never believed in value added by the manufacturing sector. The Physiocrats could not reason that the price difference between processed and unprocessed goods was made possible because of effort from somewhere. The classical scholars discussed factor intensities and factor intensity reversal in the factor proportion theory while making analysis on how countries can benefit from international trade. The classical scholars recognized the importance of labour and capital in production because their argument is that the difference in value added by labour and capital between two countries will form the basis for specialization in production. However, they did not go further in quantifying the relationship between input and output.

According Mishra (2007) the notion of inputs combination to produce a given output of goods is widely attributed to Knut Wicksell between 1900 and 1901. Mishra hints that Wicksell made the first functional relationship between production inputs and outputs, but could not apply it before the novel work of Charles Cobb and Paul Douglas. Cobb and Douglass came out with the law of production after conducting an empirical research in America manufacturing sector to show that the production of a given quantity of good depends on the combination of some factor inputs, capital and labour. They became the first group of scholars to model aggregate production function and quantify the marginal product of any factor in economic literature. From then, there emerged production function which defines a technical relationship between output and inputs in production process. As fundamentally expressed by cobb-Douglass,

$$Y = AL^{\alpha}K^{\beta} \quad (1)$$

Where Y = total output, L = units of labour, K = units of capital, and α and β are elasticity of labour and capital, and A is an efficiency parameter. Cobb-Douglass stated that the production function follows constant returns to scale, which denotes that when factor inputs are increased by 1%, output will equally increase by 1% ($\alpha+\beta = 1$).

Cobb-Douglas model received two major criticisms from scholars of economics based on its assumptions. One of the criticisms centred on assumption of constant returns to scale which the model built its analysis on. The second one and the more important is the omission of technical change, thereby believing that technology remained the same within the period of the study. Fraser (2002) regarded the restriction imposed in the model without relevant tests as econometrically unacceptable and their belief that technology was constant as very difficult to accept. However, despite the criticisms of the Cobb-Douglass production function, it is still important in the theory of production today.

The cobb-Douglass production function is very important to the business firm because it helps the firm to make rational decision on the quantity of each factor inputs to employ so as to minimize the production cost. The firm as a rational economic agent needs information on the marginal productivity of factors to be able to produce at optimum. In trying to maximize profit or minimize loss, firms can substitute one factor for another under a cost outlay. For instance if production is capital intensive, but the cost of capital is rising, decision can be taken to adjust inputs combination if rental price of labour is unchanged. More labour can be combined with less capital as long as they are the only factors of production.

Today, many other production functions have emerged to take care of some shortcomings in the cobb-Douglass model. Prominent among them are the Leontief input-output model, Harold-Domar model and constant elasticity of substitution (CES). The CES model is a production function like the cobb-Douglass model which allows any good to be produced with only two factor inputs. Moreover, it assumes CES between the two factor inputs. The drawbacks of the CES model are two. The first is that the model will break down when production of goods involves the use of more than two inputs, and number two, assumption of CES between two factor inputs is difficult to obtain (Mishra, 2007; Uzawa, 1962).

The input-output model developed by Wassily Leontief is noted as the simplest of the production function developed to explain the relationship between input and output. The production function is based on factor proportion. In the Leontief's model, when two inputs that are not substitutable are combined in a fixed proportion to produce a given output of good, increasing one of the inputs and holding the other constant in the next round of production will not change the level of output. For example, if 2 bags of sugar and 5 tons of flour are the right input combination to produce a given quantity of bread, increasing the bags of sugar to 3 while the tons of flour is unchanged will not increase the number of loaves of bread produced. The model has been adjudged the simplest of all the models developed to explain the relationship between input and output.

3. RELATED LITERATURE

Cobb-Douglass production function shows different arrays of efficient production method facing a firm depending on the output level the firm wants to attain. The relationship it expresses reveals the maximum amount of output possible from a particular input combination. Arrow et al. (1961) did not support the notion of constant returns to scale because factors of production are never substituted on a constant proportion. Other arguments and counter arguments have persisted on the validity of the aggregate production function as developed by Cobb-Douglass and others (Felipe and Adams, 2005; Robinson, 1954; Hall, 1998). The argument of Robinson (1954) is that there is a mis-education in production function that assumes all workers are alike. Making the same line of argument, Hall (1998) contends that specifying production function is always confronted with making a choice of the right algebraic form of argument to follow. In a study he carried on crop yield, he identified that there is a cost to misspecification of production function when analysing the relationship between input and output.

Moreover, Felipe and McCombie (2001) noted the difficulty of aggregation of different kinds of commodities produced in a country into a single output as assumed in Cobb-Douglass model. Their argument is that production function is supposed to be in the framework of microeconomy. In line with this argument, Guerrien and Gun (2015) also doubt the possibility of aggregating quantities of different kinds of commodities an economy can produce which will be called single output. Their position is that such assumption is difficult to believe.

However, some scholars saw important contribution of Cobb-Douglass model in economic literature. Mishra (2007) stressed that Cobb-Douglass model has made useful contribution in the area of competitive equilibrium. Hong (2008) views Cobb-Douglass model as a good description of production method. Adetunji et al. (2012) accept the wide use of the production function in works relating to productivity of factors and growth today. In a similar note, Biddle (2012) sees the Cobb-Douglas model as very innovative for the reason that it shows that statistical method can be used to derive empirical relationship between input and output. Moreover, Hagendorf (2013) accepts that despite the fact that the capitalists are interested in profit, the Cobb-Douglass production function can be used by the socialist planner.

Like every other theory, empirical tests have not produced uniform result in the Cobb-Douglass case. Fraser (2002) tried to replicate Cobb-Douglass's result using data of USA, Massachusetts, New South Wales, Victoria and New Zealand as before. He subjected the data to econometric tests and came out with a result that apart from New Zealand, result from USA, Massachusetts, New South Wales and Victoria could hardly support laws of proportion as propounded by Cobb-Douglass. Study by Raval (2011) falls in line with the Fraser's finding. Raval in a micro study with data from the manufacturing sector in America found that the capital share in the cost of production varies from time to time and it is never constant as predicted by Cobb-Douglass model.

Study by Duffy and Papageorgio (2000) equally rejects the validity of the model as a good specification of aggregate production function in a panel of 82 countries. Evidence from their panel analysis in middle and low income countries points out that the CES and the Cobb-Douglass production function may not be the correct aggregate production function. The finding in New Zealand by Szeto (2001) is similar to the result of Duffy and Papageorgio. Szeto discovered that there is substitution between primary factors (value added) and import, and thus, rejected the Cobb-Douglass specification. Other studies that invalidate the Cobb-Douglas model are Yusi (2016) in a study of the Pineapple production in in Southern Indonesia, and Hossain and Al-Amri (2010) in the manufacturing sector of Oman. Both Yusi and Hossain and Al-Amri discover increasing return to scale since output more than double whenever input is doubled. In Romania, Silaghi and Medesfalean (2014) reveal decreasing return to scale, as doubling of input leads to less than double in output.

On the other hand, there are research evidences in support and in validation of the Cobb-Douglass model. They include Chisasa and Makina (2013) in South Africa, Adetunji et al. (2012) in Nigeria and Ahmad and Khan (2015) in Pakistan. All these researches found validity in constant returns to scale property of the Cobb-Douglass production function. In Nigeria for instance, Adetunji et al. conclude that the reliance on the Cobb-Douglass model for policy will not hurt the economy. However, a contradicting result was found in Nigeria in a study by Abidemi (2010). Abidemi discovered that as the banking sector doubles its input use, its output more than doubles, suggesting increasing returns to scale in the banking sector in Nigeria.

In the case of Nigeria, the work of Abidemi is a sectoral analysis which may be the reason for the display of increasing returns to scale. Private sector production is known to be associated with efficiency. On the other hand, the work of Adetunji is a macro analysis but has a problem of specification. Adetunji et al. (2012) used capital expenditure of the government as the measure of the economy's capital. The use of only government investment as a measure of the economy's capital is grossly inadequate because it will underestimate an economy's capital and the contribution of capital to total output. To overcome the problem, we used GFCF. Moreover, since the two studies used restricted least squares, we applied the unrestricted least squares so as to see if the Cobb-Douglas model can still fit a macroeconomy data in the absence of restriction.

4. DATA AND METHODOLOGY

The Cobb-Douglas model was based on the assumption of constant returns to scale, implying that in the production decision, whenever the inputs used to produce a given output of goods is doubled, total output will automatically double. This restriction imposed in the model has been the centre of controversy among economists (Bhanumurthy, 2002; Antras, 2004; Rana et al., 2010). In a study using America data, Antras discovers that under restriction, the cobb-Douglas production model fits America data. However, if the restriction is removed, it cannot describe America economy well. Other criticism of the model include problem of specification (Hossain et al., 2012), and identification (Mairesse, 2005). The criticisms of the Cobb-Douglas model compelled Hossain et al. (2012) to make a modification of the model in a study in the manufacturing sector in Bangladesh. The Cobb-Douglas production model is important for developing countries as the model can help answer the question of right input combination to obtain maximum output.

We used quarterly data generated from Central Bank of Nigeria Statistical Bulletin (2012) between 1990 and 2012. High frequency data is preferred in this case because production and investment is a daily affair and loss of information will be less in quarterly data than the annual ones. That is, production decision is taken at time interval less than a year (Zellner et al., 1966).

Restating the typical Cobb-Douglass production function in a stochastic form,

$$Y = AL^{\alpha}K^{\beta}e^{ui} \quad (2)$$

Where,
Y = Total output,
L = Units of labour,
K = Units of capital, and α and β are elasticity of labour and capital, and A is an efficiency parameter while e is the base of natural logarithm and ui is the stochastic disturbance term.

From a priori expectation, one of the following conditions can hold after estimation under the Cobb-Douglas restriction, viz.:

$\alpha+\beta>1$

$\alpha+\beta<1$

$\alpha+\beta = 1$

If the estimated function result is such that $\alpha+\beta = 1$, Cobb-Douglass hypothesis is validated, implying constant return to scale.

Linearizing 2,

$$\ln Y_t = \ln A+\alpha \ln L_t+\beta \ln K_t+\mu_t \quad (3)$$

$$\ln Y_t = \lambda+\alpha \ln L_t+\beta \ln K_t+\mu_t (\ln A = \lambda) \quad (4)$$

L is the labour force participation, K= gross fixed capital formation (GFCF) which is an important departure from the work of Adetunji et al. (2012) who used government capital investment, α and β are coefficients to be estimated, ln = natural logarithm operator, and t = time horizon.

5. UNRESTRICTED LEAST SQUARE

Unrestricted least squares assume that the only existing information is information within the parameters of the model. It estimates the parameter of an equation without taking into account the linear equality restriction, that is, $\alpha+\beta = 1$ (Adetunji et al., 2012). Unrestricted least square uses the t-statistic instead of the f-statistic. This involves conducting a test hypothesis after estimating α and β using the t-test, guided by the test statistic stated below:

$$to = \frac{(\pm+^2)-1}{\sqrt{var(\alpha)+var(\beta)+2cov(\alpha,\beta)}} \quad (5)$$

From 5,
$H_o = t_{cal} < t_{tab}$, Cobb-Douglas model of constant returns to scale is validated
$H_1 = t_{cal} > t_{tab}$, Cobb-Douglas model of constant returns to scale is not validated.

6. RESULTS

6.1. Unit Root Test

Test of stationarity of time series data is indispensable if the research is to add value to literature since unstationary data leads to spurious result in econometric. The result of the unit root test is presented in Table 1.

Regression result presented in Table 2 can be presented in linear form as modelled in equation (4). Therefore,

$$d\text{-}\ln Y = 0.003080 + 0.497442L + 0.077108K + e_t \quad (6)$$

(Se) = (.002326) (0.062357) (0.017153)

(t) = (1.32) (7.98) (4.5)

Cov (α, β) = (0.3660)

By fitting Cobb-Douglass production in its exponential form, equation 5 leads to:

$$Y = 0.00308L^{0.4970} K^{0.0771} \quad (7)$$

From 7, under its restricted assumption,

Table 1: Result of unit root test

Variable	ADF test statistics	5% level	Order of integration
d-ln Y	−16.171	−3.460	1 (1)
d-ln L	−11.957	−3.461	1 (1)
d-ln K	11.176	−3.461	1 (1)

Source: Authors' computation

Table 2: Regression result (dependent variable d-lnGDP)

Variable	Coefficient	Standard error	t	P> /t/
d-lnL	0.497442	0.062357	7.98	0.0000
d-lnK	0.077108	0.017153	4.5	0.0000
λ=C	0.003080	0.002326	1.32	0.1890
SE of regression	0.020382			
Sum squared residual	0.036558			
Log likelihood	226.6735			
Mean dependent variable	0.013194			
Standard deviation dependent variable	0.031512			

Source: Authors' computation. R^2=0.59; Adjusted R^2=0.58; DW=2.05; F-stat=63.57; Prob F-stat=0.0000

$\alpha+\beta = 0.497442+0.077108=0.575$

Therefore, $\alpha+\beta<1$, implying decreasing returns to scale.

However, since we are using the unrestricted approach, equation (5) applies.

6.2. Examination of the Validity of Cobb-Douglass Production Function within Unrestricted Least Squares

Removal of the constraint imposed by the Cobb-Douglas model implies that our decision is not to be guided by the outcome of $\alpha+\beta$. Conclusion on the validity of the function will be based on the t-test of equation (4), and depends on whether $t_{cal} > t_{\alpha/2}$ (n-k), where n = number of observations, k = number of parameters estimated in the equation, and α is the 5% level of significance. From Table 2, $t_{\alpha/2}$ (n-k) = 2.0.

Similarly, substituting the values in Table 2 into equation (5), $t_{cal} = -0.4994$. This is because the variance of α and β is the square of their standard error.

Since absolute value of t_{cal} = 0.4994 and $t_{\alpha/2}$(n-k) = 2.0, $t_{cal} < t_{tab}$ (0.4994<2.0) in absolute term. It entails accepting the null hypothesis of constant return to scale in Nigeria economy. Therefore, the Cobb-Douglass production function of constant returns to scale is valid for Nigeria economy. On the other hand, the Cobb-Douglass model fits well in Nigeria economy. For this reason, whenever inputs are doubled in production process, output will double as well in Nigeria. The finding falls in line with the earlier study done in Nigeria by Adetunji et al. (2012) using restricted lest squares, but different from the study by Abidemi. This has implication for economic policy and management in Nigeria. For one, when an analysis is made using aggregate economy data in a developing country like Nigeria, the public sector inefficiency will outweigh private sector efficiency. Two, whenever an economy is public sector driven as in the case of Nigeria, increase in public sector investment without considerable increase in private sector investment will not be enough to increase factor productivity.

7. CONCLUSION

The study has gone through the rigours of unrestricted least squares estimation procedure to test the fit of Cobb-Douglass production function in Nigeria. Two important factors motivated the study in Nigeria. One, Nigeria economy has continued to perform poorly for more than two decades and a half despite continuous investment by both the private and public sector. Two, the use of only capital investment of the public sector in the work of Adetunji et al. (2012) was seen as inadequate to explain efficiency of capital in Nigeria. Government investment from experience is less efficient in the developing countries compared to private ones. Unfortunately, despite important departures from the approach of existing study, we have arrived at the same result that even if the restriction imposed by the cobb-Douglass production is removed, the model fits well in Nigeria economy. The result is consistent with the finding in Czech economy by Hajkova and Hurnik (2007). However, it is in opposite direction with the finding of Antras (2004) which shows that under its restricted form, Cobb-Douglas model fits American economy well and if the restriction is removed, it will no more fit the economy.

Finally, research findings are trying to suggest that when it comes to the use of Macro data, Cobb-Douglas model is valid, but with the use of micro data, it will be invalidated (Raval, 2011; Abidemi, 2010).

REFERENCES

Abidemi, A. (2010), Capital-labour substitution and banking sector performance in Nigeria (1960-2008). Central Bank of Nigeria Economic and Financial Review, 48(2), 109-130.

Adetunji, A., Ibraheem, A., Ademuyiwa, J.A. (2012), On restricted least square: The cobb-douglass production function for Nigerian economy. Developing Countries Studies, 2(8), 68-73.

Antras, P. (2004), Is the U.S. aggregate production function cobb-douglas? New estimates of the elasticity of substitution. Contributions to Macroeconomic, 4(1), 1-34.

Arrow, K.J., Chenery, H.B., Minhas, B.S., Solow, R.M. (1961), Capital-labour substitution and economic efficiency. The Review of Economics and Statistics, 47(3), 225-250.

Bhanumurthy, K.V. (2002), Arguing a case for the cobb-douglas production function. Review of Commerce Studies, 2002, 75-91.

Biddle, J. (2012), The Introduction of the cobb-douglas regression. Journal of Economic Perspective, 26(2), 223-236.

Central Bank of Nigeria. (2012), Statistical Bulletin. CBN.

Chisasa, J., Makina, D. (2013), Bank credit and agricultural output in South Africa: A cobb-douglass empirical analysis. International Business and Economics Research Journal, 2(4), 387-398.

Duffy, J., Papageorgio, C. (2000), Cross-country empirical investigation of aggregate production function specification. Journal of Economic Growth, 5, 87-120.

Felipe, J., Adams, F.G. (2005), 'A theory of Production' The estimation of cobb-douglass function: A retrospective View. Eastern Economic Journal, 31(3), 427-445.

Felipe, J., McCombie, J.S.L. (2001), How Sound are the Foundations of Aggregate Production Function? Economics Discussion Papers

Series No. 116. University of Otago.
Fraser, I. (2002), The cobb-douglass production function: A antipodean defence? Part 1. Economic Issues, 7, 39-58.
Guerrien, B., Gun, O. (2015), Putting an end to the aggregate function of production forever? Real-World Economic Review, 73(2015), 99-109.
Hagendorf, K. (2013), The Cobb-Douglass Production Function and Political Economy. Available from: http://www.ssrn.com/abstract=1541468.
Hajkova, D., Hurnik, J. (2007), Cobb-douglas production function: The case of a converging economy. Czech Journal of Economics and Finance, 57(9-10), 465-476.
Hall, H.H. (1998), Choosing an Empirical Production Function: Theory, Nonnested Hypotheses, Costs of Specifications, Agricultural Economics. Research Report No. 59.
Hong, T.B. (2008), Cobb-douglas production function. In: Stewart, J., editor. Calculus: Early Transcendentals. 6thed. United States, Australia, Singapore, Canada, Australia, United Kingdom, Brazil: Thomson Brooks/Cole/.p857-887.
Hossain, M., Majumder, A.K., Basak, T. (2012), An application of non-linear cobb-douglas production function to selected manufacturing industries in Bangladesh. Open Journal of Statistics, 2, 460-468.
Hossain, M.Z., Al-Amri, K.S. (2010), Use of Cobb-Douglas Production Model on Some Selected Manufacturing Industries in Oman. Available from: http//www.emeraldinsight.com/1753-7983.
Mairesse, J. (2005), Panel-data estimates of the production function and the revenue function: What difference does it make? Scandinavian Journal of Economics, 107(4), 651-672.
Miller, E. (2008), An Assessment of CES and Cobb-Douglass Production Functions. Available from: http://www.cbo.gov.
Mishra, S.K., editor. (2007), A Brief History of Production Function. Munich Personal RePEc Archives (MPRA, 5254).
Rana, M., Baten, A., Kamil, A.A. (2010), A stochastic frontier approach for empirical tests of efficiency wage models. Scientific Research and Essays, 5(11), 1234-1242.
Raval, D. (2011), Beyond cobb-douglass: Estimation of CES production function with factor augmenting technology. Centre of Economic Studies, 2011, 5-11.
Robinson, J. (1953-1954), The production function and the theory of capital. Review of Economics Studies, 21(2) 81-106.
Silaghi, M.P., Medesfalean, R. (2014), Some insights about determinants of economic growth in Romania: An empirical exercise. Theoretical and Applied Economics, 21(6), 23-36.
Szeto, K.L. (2001), An Econometric Analysis of a Production Function foe New Zealand, Treasury Working Paper No. 01/31.
Uzawa, H. (1962), Production functions with constant elasticities of substitution. Review of Economic Studies, 29,291-299.
Yusi, M.S. (2016), The analysis of production function and farm marketing efficiency of pineapple (*Ananas comosus* L Merr) in south Sumatera province, Indonesia. Journal of Agricultural Research, 11(23), 1990-1998.
Zellner, A., Kmenta, J., Dreze, J. (1966), Specification and estimation of cobb-douglas production function models. Econometrica, 34(4), 784-795.